A Technology-Gap Model of ‘Premature’ Deindustrialization

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Updated on 2024-04-15; 11:25 PM

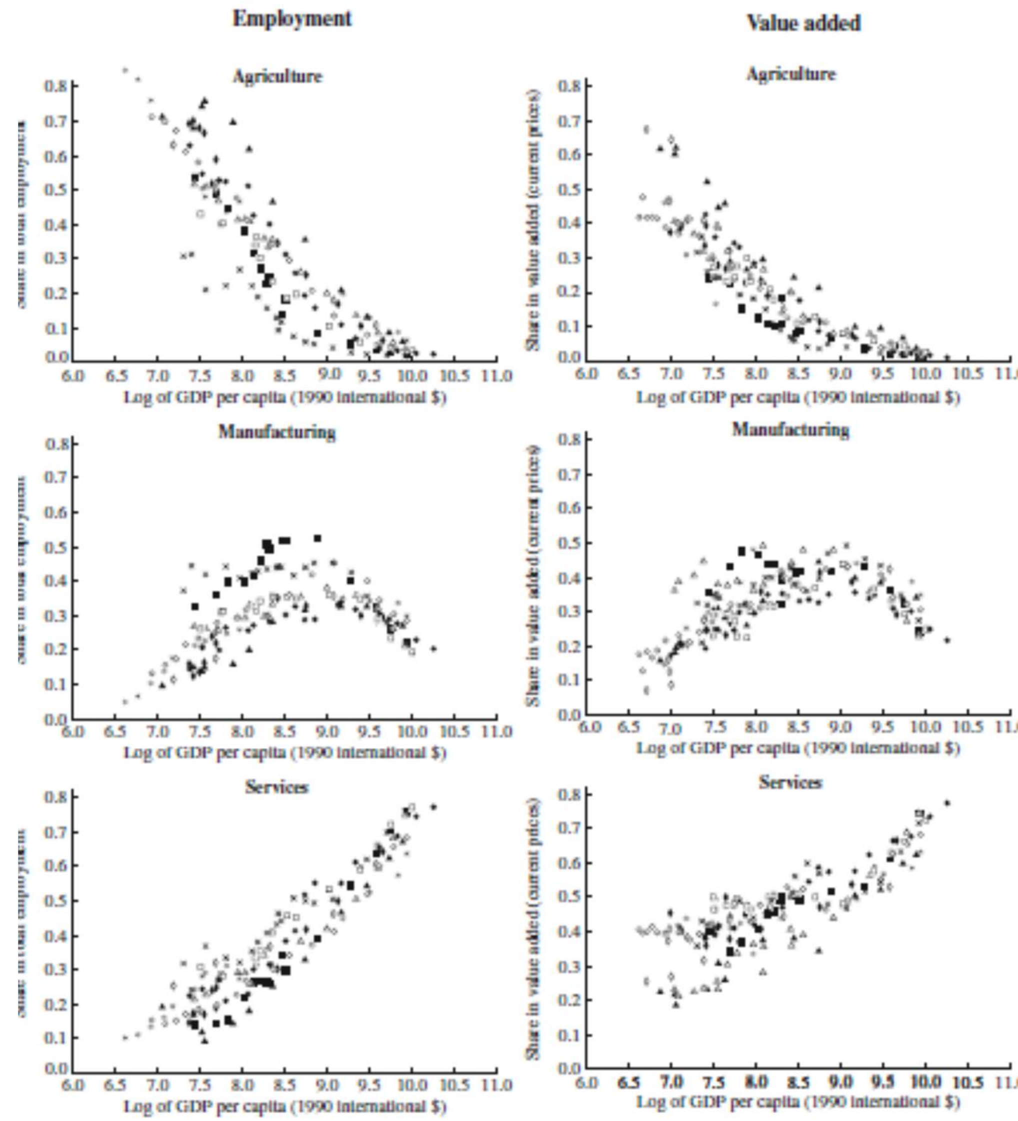
Teaching Slides

Introduction

Structural Change

As per capita income rises, the employment or value-added shares

- *Fall* in Agriculture
- *Rise* in Services
- *Rise and Fall* in Manufacturing



From Herrendorf-Rogerson-Valentinyi (2014)

Evidence from Long Time Series for the Currently Rich Countries (Belgium, Finland, France, Japan, Korea, Netherlands, Spain, Sweden, United Kingdom, and United States) 1800-2000

Premature Deindustrialization (PD): Rodrik (JEG 2016)

Late industrializers reach their M-peak and start deindustrializing

- *Later* in time
- *Earlier* in per capita income
- with the *lower* peak M-sector shares,

compared to early industrializers.

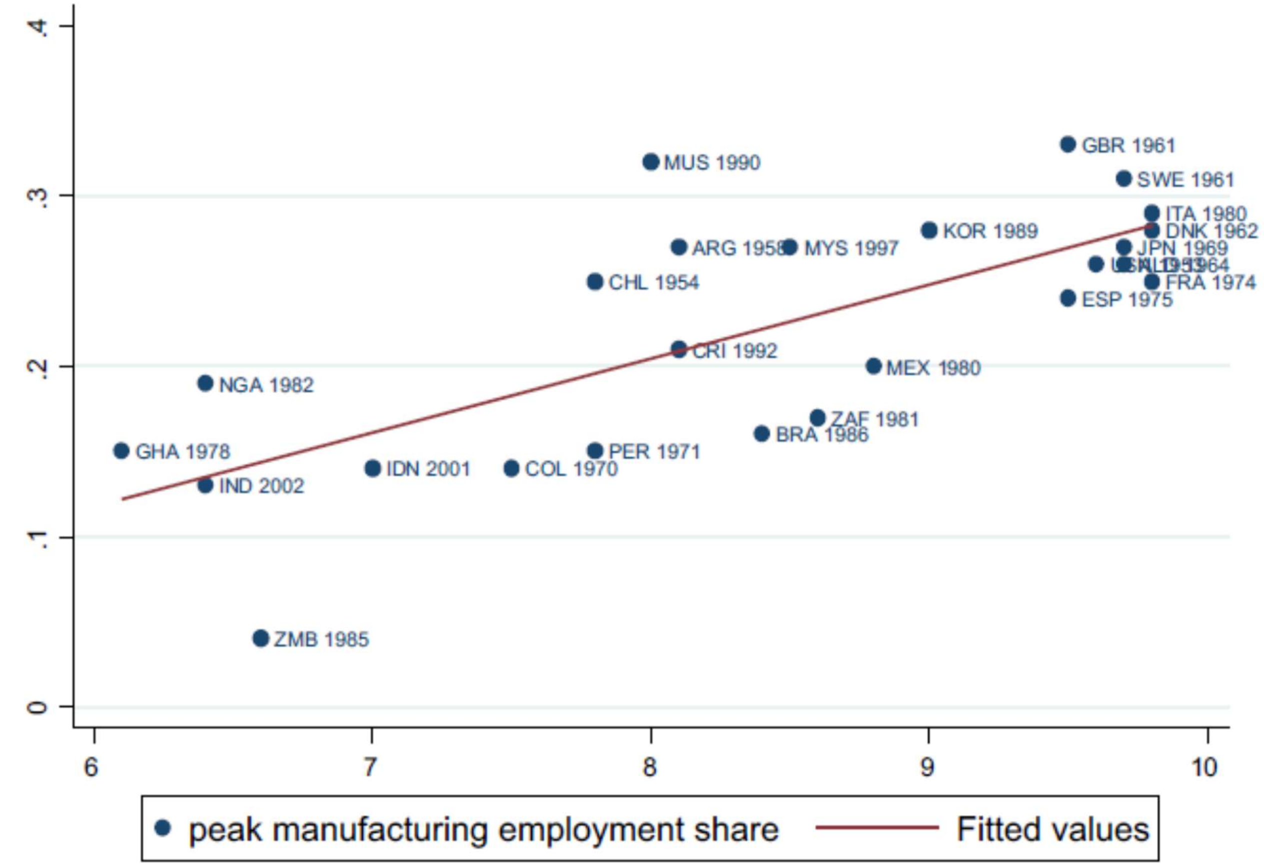


Fig. 5 Income at which manufacturing employment peaks (logs)

Rodrik (2016) focuses on documenting the patterns, without offering a causal explanation or making normative statements. But

- He speculates that globalization may be a cause.
- He cautions against drawing policy implications, but the word, "premature," may seem to suggest some types of inefficiency.

In our proposed mechanism,

- PD occurs in the *efficient* equilibrium of a *closed* economy.
- PD is robust to opening up for trade but weakened.

This Paper: A Parsimonious Model of Premature Deindustrialization (PD)

3 Goods/Sectors: 1=(**A**)griculture, 2=(**M**)anufacturing, 3=(**S**)ervices, *homothetic CES with gross complements* ($\sigma < 1$)

Frontier Technology: $\bar{A}_j(t) = \bar{A}_j(0)e^{g_j t}$, with $g_1 > g_2 > g_3 > 0$ ⇒ a decline of A, a rise of S, and a hump-shaped of M in each country through **the Baumol (relative price) effect**, as in Ngai-Pissarides (2007)

Actual Technology Used: $A_j(t) = \bar{A}_j(t - \lambda_j)$ due to **Adoption Lags,** $(\lambda_1, \lambda_2, \lambda_3)$.

$$A_j(t) = \bar{A}_j(t - \lambda_j) = \bar{A}_j(0)e^{-g_j \lambda_j} e^{g_j t} \implies \frac{\partial}{\partial \lambda_j} \ln\left(A_j(t)\right) = -g_j < 0$$

λ_j has **no "growth" effect, but negative "level" effects,** $e^{-\lambda_j g_j}$, amplified by g_j.

Log-submodularity: g_j magnifies the (negative) impact of the adoption lag on productivity: $\frac{\partial}{\partial g_j}\left(\frac{\partial}{\partial \lambda_j} \ln e^{-g_j \lambda_j}\right) < 0$

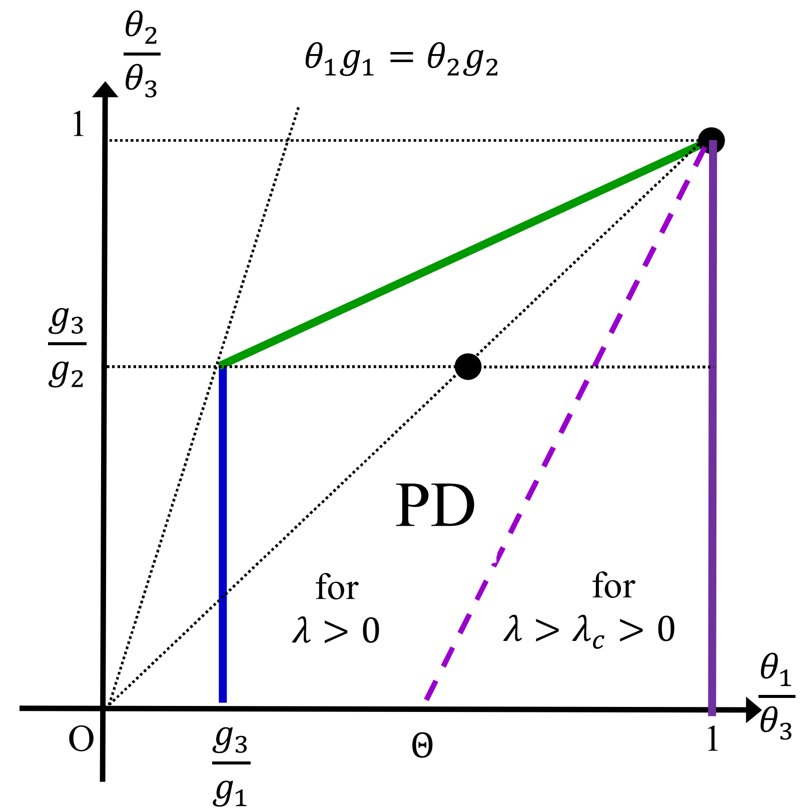
One-dimension of cross-country heterogeneity: For $(\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3)\lambda$,
- $\lambda \geq 0$, **Technology Gap,** country-specific, as in Krugman (1985); their ability to adopt the frontier technologies.
- $\theta_j > 0$: sector-specific, unlike Krugman (1985); how much λ affects the adoption lag and productivity in each sector.

$$A_j(t) = \bar{A}_j(0)e^{-g_j \theta_j \lambda} e^{g_j t} \implies \frac{\partial}{\partial \lambda} \ln\left(\frac{A_j(t)}{A_k(t)}\right) = -(\theta_j g_j - \theta_k g_k).$$

Main Results: Conditions for PD, defined as "A high-λ country reaches its peak later in time, with lower peak M-share at lower peak time per capita income."

i) $\theta_1 g_1 > \theta_3 g_3$: cross-country productivity difference larger in A than in S.
High relative price of A/low relative price of S in a high-λ country causes a delay.

ii) $\left(1-\frac{g_3}{g_1}\right)\left(\frac{\theta_2}{\theta_3}-1\right)+\left(1-\frac{g_3}{g_2}\right)\left(1-\frac{\theta_1}{\theta_3}\right)<0$:
Technology adoption takes not too long in M.
Not too high relative price of M in a high-λ country keeps the M-share low.
Under the above conditions,
iii) $\theta_1 < \theta_3$: Technology adoption takes longer in S than in A.
Longer adoption lag in S in a high-λ country causes "premature" deindustrialization.

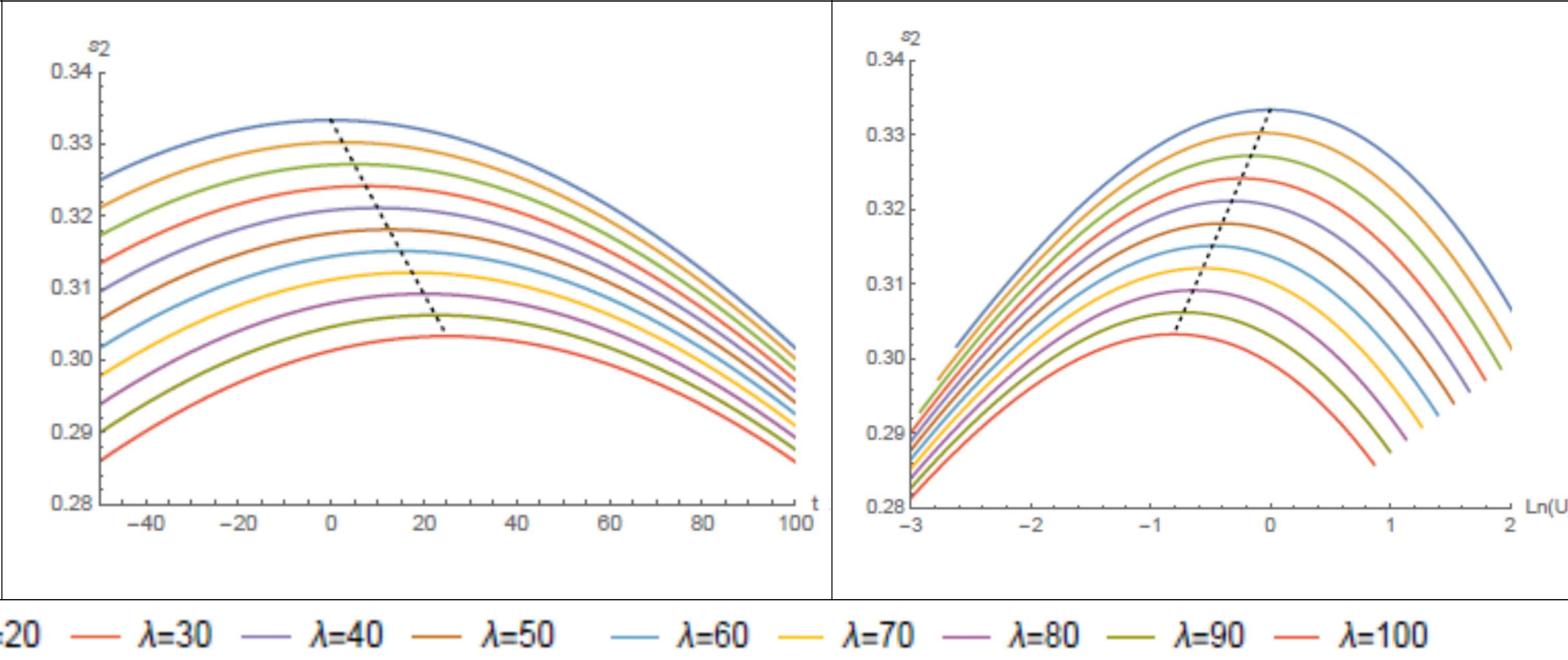


Some Implications
No PD if $\theta_1 = \theta_2 = \theta_3$. Latecomers would follow the same path with a delay.
i) & ii) $\Rightarrow \theta_1 g_1 > \max\{\theta_2 g_2, \theta_3 g_3\}$: Cross-country productivity difference is the largest in A.
$\theta_2 g_2 - \theta_3 g_3$ can be either positive or negative;
slightly negative when calibrated to match Rodrik's (2016; Table 10) findings.

A Numerical Illustration.

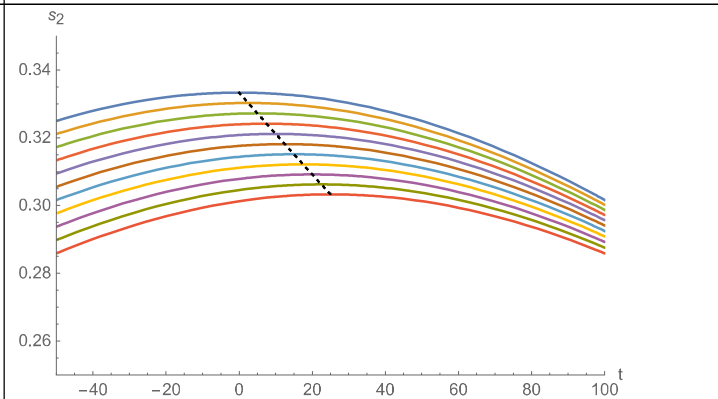
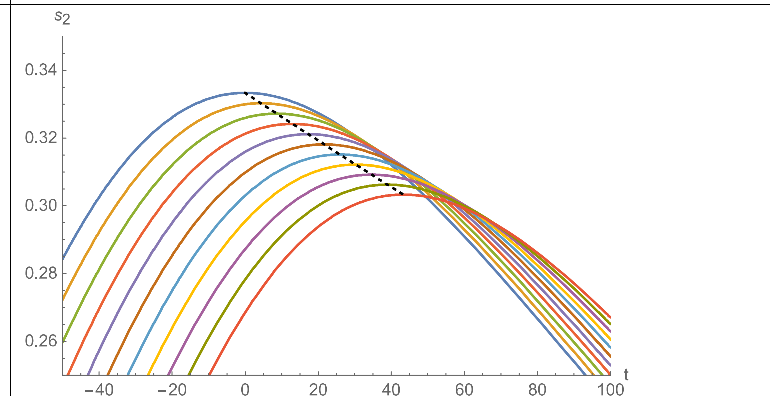
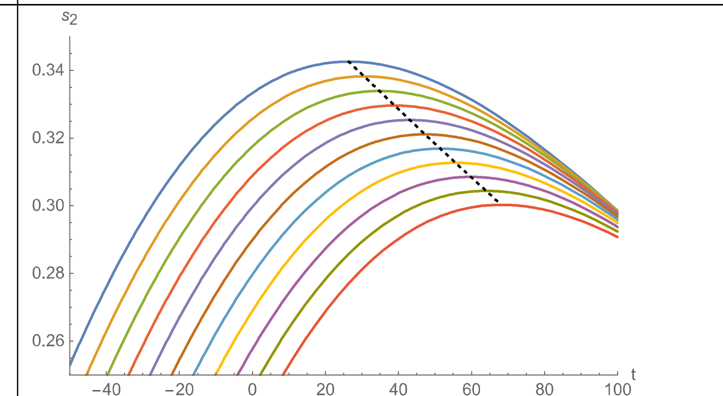
$\theta_1 = \theta_2 < \theta_3 = 1$ with $g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%$; $\sigma = 0.6$; Labor share $= 2/3$. We set the other parameters, w.l.o.g., so that the peak time, $\hat{t} = 0$ and the peak time income per capita, $U(\hat{t}) = 1$ if $\lambda = 0$.

Example 2a	$\left(t, s_2(t)\right)$	$\left(\ln U(t), s_2(t)\right)$
$\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.5 = \frac{g_3}{g_2}$ $\Longrightarrow \theta_1 g_1 > \theta_2 g_2 = \theta_3 g_3$		



1st Extension: Adding **the Engel Effect** with **Nonhomothetic CES** (a la Comin-Lashkari-Mestieri)

Nonhomotheticity changes the shape of trajectories greatly, but not on how technology gaps, λ, affects the peak values.

<table>
<tr><th></th><th>Homothetic case ($\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$)</th><th>Unbiased case($\varepsilon_1 = .4 < \varepsilon_2 = 1 < \varepsilon_3 = 1.6$)</th><th>Biased case($\varepsilon_1 = .4 < \varepsilon_2 = 1.2 < \varepsilon_3 = 1.4$)</th></tr>
<tr><td>$(t, s_2(t))$</td><td>
</td><td>
</td><td>
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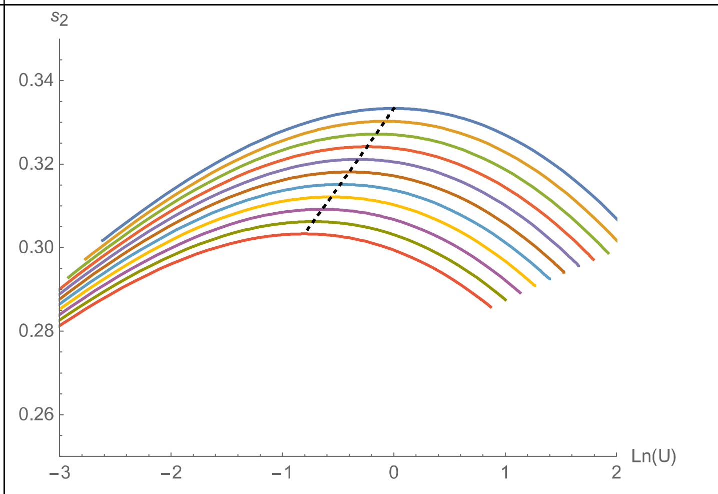
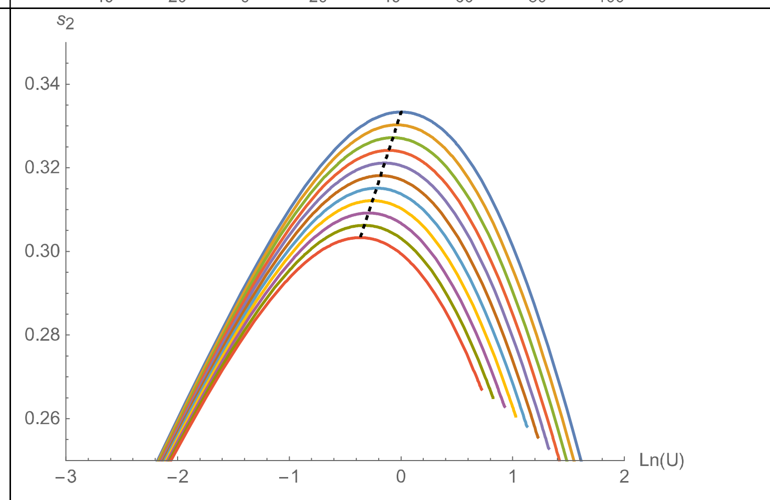
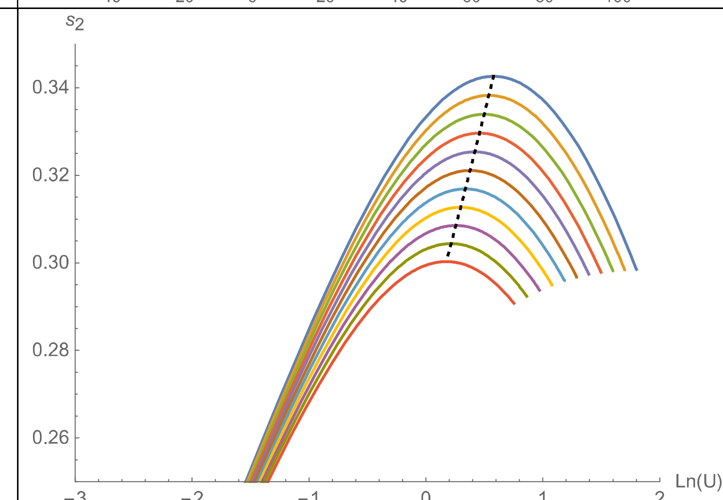
<tr><td>$(\ln U(t), s_2(t))$</td><td>
</td><td>
</td><td>
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We also show that the Engel effect *alone* could not generate PD *without counterfactual implications*.

2nd Extension: International Trade

One implication of our mechanism for PD (consistent with the empirical evidence):

$$\frac{\partial}{\partial \lambda} \ln\left(\frac{A_1(t)}{A_2(t)}\right) = -(\theta_1 g_1 - \theta_2 g_2) < 0.$$

- A low-λ country has comparative advantage in A and a high-λ country has comparative advantage in M.
- Opening up for trade allows a high-λ country to export M to a low-λ country.
- Our mechanism for PD is weakened by opening up for trade, but PD continues to hold, as long as the trade cost is not too small.
- Consistent with the findings that East Asia “suffers” less from PD (Rodrik 2016).

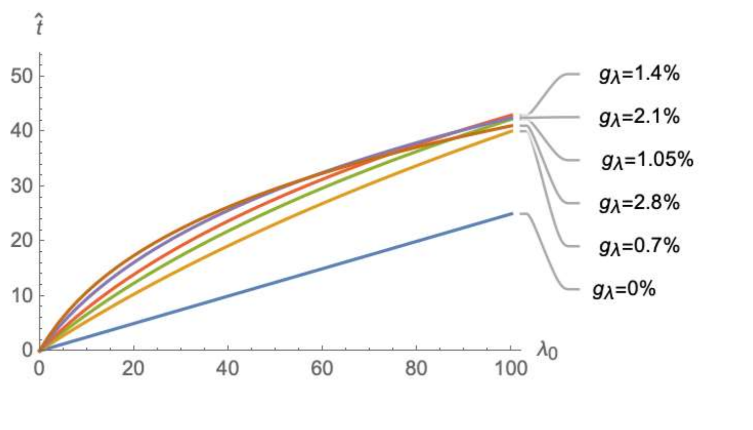
Under our mechanism, PD occurs *not because of*, but *in spite of* international trade.

3rd Extension: Introducing Catching-up

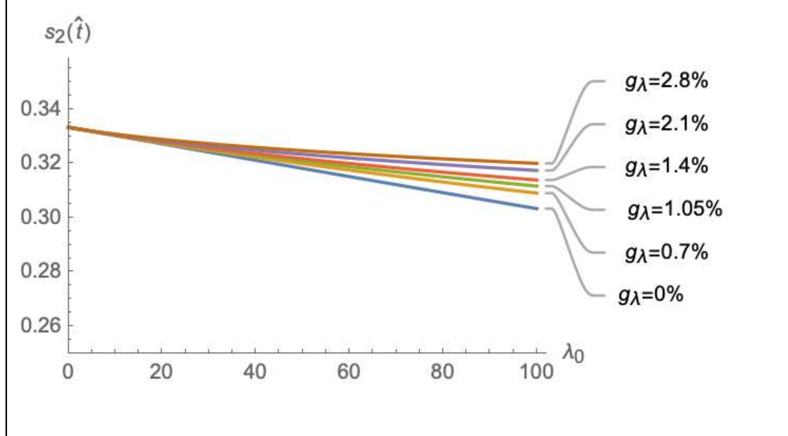
$$A_j(t) = \bar{A}_j(0)e^{g_j(t-\theta_j\lambda_t)}, \quad \text{where} \quad \lambda_t = \lambda_0 e^{-g_\lambda t},$$

Countries differ only in the *initial* value, $\boldsymbol{\lambda_0}$, converging exponentially over time at **the same rate,** $g_\lambda > 0$

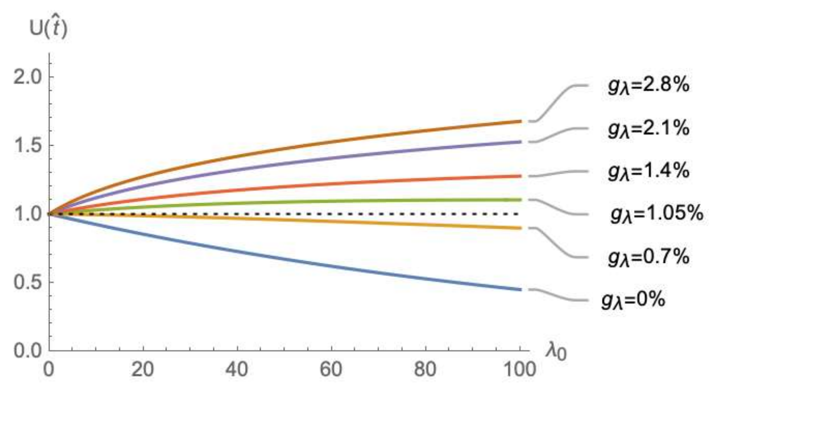
Peak Time



Peak M-Share



Peak time Per Capita Income



Higher-λ countries

- peak later in time,
- have lower peak M-shares
- have lower peak time per capita income, unless g_λ is too large.

(Very Selective) Literature Review. Herrendorf-Rogerson-Valentinyi (14) for a survey on structural change.

Related to The Baseline Model

Premature Deindustrialization, Dasgupta-Singh (06), Palma (14), **Rodrik (16)**
The Baumol Effect: Baumol (67), **Ngai-Pissarides (07),** Nordhaus (08)
Cross-country heterogeneity in technology development

- *Distance to the frontier:* **Krugman** (85), Acemolgu-Aghion-Zilibotti (06)
- *Log-supermodularity:* **Krugman** (85), Matsuyama (05), Costinot (09), Costinot-Vogel (15)
- *Productivity difference across countries the largest in A:* Caselli (05), Gollin et.al. (14, AERP&P)
- *Small adoption lags in M;* Rodrik (2013)

Related to Three Extensions

The Engel Effect (Nonhomotheticity); Murphy et.al. (89), Matsuyama (92,02), Kongsamut et.al. (01), Foellmi-Zweimueller (08), Buera-Kaboski (09,12), Boppart (14), **Comin-Lashkari-Mestieri (21),** Matsuyama (19), Lewis et.al. (21), Bohr-Mestieri-Yavuz (21)
Open Economy Implications: Matsuyama (92,09), Uy-Yi-Zhang (13), Sposi-Yi-Zhang (19), Fujiwara-Matsuyama (WinP)
Catching-Up/Technology Diffusion: Acemoglu (08), Comin-Mestieri (18)

The Issues We Abstract From

Sector-level productivity growth rate differences across countries: Huneeus-Rogerson (20)
Endogenous growth, externalities, Matsuyama (92).
Sectoral wedges/misallocation: Caselli (05), Gollin et.al. (14 QJE) and many others
Nominal vs. Real expenditure; Employment vs. Value Added shares; Compatibility with aggregate balance growth, investment vs consumption, sector-specific factor intensities, skill premium, home production, productivity slowdown, etc.

Structural Change, the Baumol Effect, and Adoption Lags

Three Complementary Goods/Competitive Sectors, $j = 1, 2, 3$

Sector-1 = **(A)**griculture, Sector-2 = **(M)**anufacturing, Sector-3 = **(S)**ervices.

Demand System: L Identical HH, each endowed with 1 unit of mobile labor, earning the wage w & κ_j units of managerial skills, specific to j, each earning the rent, ρ_j.

Budget Constraint:

$$\sum_{j=1}^{3} p_j c_j \leq E \equiv w + \sum_{j=1}^{3} \rho_j \kappa_j = \frac{1}{L} \sum_{j=1}^{3} p_j Y_j$$

CES Preferences:

$$U(c_1, c_2, c_3) = \left[\sum_{j=1}^{3} (\beta_j)^{\frac{1}{\sigma}} (c_j)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

with $\beta_j > 0$ and $0 < \sigma < 1$ (gross complementarity)

Expenditure Shares:

$$m_j \equiv \frac{p_j c_j}{E} = \beta_j \left(\frac{p_j}{P}\right)^{1-\sigma}; \quad P = \left[\sum_{k=1}^{3} \beta_k (p_k)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

Real Per Capita Income

$$U = \frac{E}{P} = \left[\sum_{k=1}^{3} \beta_k \left(\frac{E}{p_k}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}.$$

Three Competitive Sectors: Production

Cobb-Douglas

$$Y_j = \tilde{A}_j(\kappa_j L)^{\alpha}(L_j)^{1-\alpha} = A_j(L)^{\alpha}(L_j)^{1-\alpha} = LA_j(s_j)^{1-\alpha}$$

where $A_j \equiv \tilde{A}_j(\kappa_j)^{\alpha}$. $\alpha \in [0,1)$: the span of control parameter, which introduces diminishing returns in labor.

Labor Share $\frac{wL_j}{p_jY_j} = 1-\alpha$ **Profit (Managerial Rent) Share** $\frac{\rho_j\kappa_j L}{p_jY_j} = \alpha$

Sectoral Share in Employment

$$s_j \equiv \frac{L_j}{L}; \qquad \sum_{j=1}^{3} s_j = 1$$

Sectoral Sector in Value-Added

$$\frac{p_jY_j}{EL} = \frac{p_jY_j}{\sum_{k=1}^{3} p_kY_k}$$

$$\frac{p_jY_j}{EL} = s_j = \left(\frac{p_jA_j}{E}\right)^{1/\alpha}; \; E = \left[\sum_{k=1}^{3}(p_kA_k)^{\frac{1}{\alpha}}\right]^{\alpha}.$$

Equilibrium: The expenditure shares are equal to the employment and value-added shares.

$$\beta_j \left(\frac{p_j}{P}\right)^{1-\sigma} = m_j = \frac{p_j Y_j}{EL} = s_j = \left(\frac{p_j A_j}{E}\right)^{1/\alpha}$$

which lead to

Equilibrium Shares

$$s_j = \frac{\left[\beta_j^{\frac{1}{\sigma-1}} A_j\right]^{-a}}{\sum_{k=1}^{3}\left[\beta_k^{\frac{1}{\sigma-1}} A_j\right]^{-a}}$$

Per Capita Income

$$U = \left\{\sum_{k=1}^{3}\left[\beta_k^{\frac{1}{\sigma-1}} A_j\right]^{-a}\right\}^{-\frac{1}{a}}$$

where

$$a \equiv \frac{1-\sigma}{1-\alpha(1-\sigma)} = -\frac{\partial \log(s_j/s_k)}{\partial \log(A_j/A_k)} > 0,$$

which captures how much relatively *high* productivity in a sector contributes to its relatively *low* equilibrium share. α magnifies this effect by increasing a.

Productivity Growth:

$$A_j(t) = \bar{A}_j(t-\lambda_j) = \bar{A}_j(0)e^{g_j(t-\lambda_j)} = \bar{A}_j(0)e^{-\lambda_j g_j}e^{g_j t}$$

$\bar{A}_j(t) = \bar{A}_j(0)e^{g_j t}$: **Frontier Technology** in j, with a constant **growth rate** $g_j > 0$.
$A_j(t) = \bar{A}_j(t-\lambda_j)$; λ_j = **Adoption Lag** in j.

- λ_j has **no "growth" effect,** but has **a negative "level" effect,** $e^{-\lambda_j g_j}$, which is proportional to g_j.

Key: Log-submodularity, $\frac{\partial}{\partial g_j}\left(\frac{\partial}{\partial \lambda_j}\ln e^{-\lambda_j g_j}\right) < 0$: g_j magnifies the negative effect of the adoption lag on productivity

✓ A large adoption lag doesn't matter much in a sector with slow productivity growth.
✓ Even a small adoption lag matters a lot in a sector with fast productivity growth.

$$U(t) = \left\{\sum_{k=1}^{3}\left[\beta_k^{\frac{1}{\sigma-1}}A_k(t)\right]^{-a}\right\}^{-\frac{1}{a}} = \left\{\sum_{k=1}^{3}\tilde{\beta}_k e^{-a g_k(t-\lambda_k)}\right\}^{-\frac{1}{a}}, \quad where\ \tilde{\beta}_k \equiv \left(\frac{\beta_k^{\frac{1}{1-\sigma}}}{\bar{A}_k(0)}\right)^{a} > 0.$$

Longer adoption lags would shift down the time path of $U(t)$.

$$g_U(t) \equiv \frac{U'(t)}{U(t)} = \sum_{k=1}^{3} g_k s_k(t)$$

The growth rate in per capita income is the weighted average of the sectoral growth rates

Relative Prices: $$\left(\frac{p_j(t)}{p_k(t)}\right)^{1-\sigma} = \left[\left(\frac{\beta_j}{\beta_k}\right)^{-\alpha} \frac{\bar{A}_j(0)}{\bar{A}_k(0)}\right]^{-a} e^{a(\lambda_j g_j - \lambda_k g_k)} e^{a(g_k - g_j)t} \Longrightarrow \frac{d \ln\left(\frac{p_j(t)}{p_k(t)}\right)}{dt} = \frac{a(g_k - g_j)}{1-\sigma}$$

Relative Growth Effect: $p_j(t)/p_k(t)$ is de(in)creasing over time if $g_j > (<) g_k$. Speed independent of λ_j and λ_k.
Relative Level Effect: A higher $\lambda_j g_j - \lambda_k g_k$ raises $p_j(t)/p_k(t)$ at any point in time.
Note: For a fixed $\lambda_j > 0$, a higher g_j makes the relative price of j higher (though declining faster).

Relative Shares: $$\frac{s_j(t)}{s_k(t)} = \frac{\beta_j}{\beta_k}\left(\frac{p_j(t)}{p_k(t)}\right)^{1-\sigma} = \frac{\tilde{\beta}_j}{\tilde{\beta}_k} e^{a(\lambda_j g_j - \lambda_k g_k)} e^{a(g_k - g_j)t} \Longrightarrow \frac{d \ln\left(\frac{s_j(t)}{s_k(t)}\right)}{dt} = a(g_k - g_j)$$

Relative Growth Effect: $s_j(t)/s_k(t)$ is de(in)creasing over time if $g_j > (<) g_k$. Speed independent of λ_j and λ_k.
Shift from faster growing sectors to slower growing sectors over time.
Relative Level Effect: A higher $\lambda_j g_j - \lambda_k g_k$ raises $s_j(t)/s_k(t)$ at any point in time.
Note: For a fixed $\lambda_j > 0$, a higher g_j makes the relative share of j higher (though declining faster).

Structural Change with the Baumol (Relative Price) Effect: Let $g_1 > g_2 > g_3 > 0$

Decline of Agriculture: $s_1(t)$ is decreasing in t, because

$$\frac{1}{s_1(t)} - 1 = \frac{s_2(t)}{s_1(t)} + \frac{s_3(t)}{s_1(t)} = \left[\frac{\tilde{\beta}_2}{\tilde{\beta}_1} e^{a(\lambda_2 g_2 - \lambda_1 g_1)}\right] e^{a(g_1 - g_2)t} + \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_1} e^{a(\lambda_3 g_3 - \lambda_1 g_1)}\right] e^{a(g_1 - g_3)t}$$

Rise of Services: $s_3(t)$ is increasing in t, because

$$\frac{1}{s_3(t)} - 1 = \frac{s_1(t)}{s_3(t)} + \frac{s_2(t)}{s_3(t)} = \left[\frac{\tilde{\beta}_1}{\tilde{\beta}_3} e^{a(\lambda_1 g_1 - \lambda_3 g_3)}\right] e^{-a(g_1 - g_3)t} + \left[\frac{\tilde{\beta}_2}{\tilde{\beta}_3} e^{a(\lambda_2 g_2 - \lambda_3 g_3)}\right] e^{-a(g_2 - g_3)t}$$

Rise and Fall of Manufacturing: $s_2(t)$ is hump-shaped in t, because

$$\frac{1}{s_2(t)} - 1 = \frac{s_1(t)}{s_2(t)} + \frac{s_3(t)}{s_2(t)} = \left[\frac{\tilde{\beta}_1}{\tilde{\beta}_2} e^{a(\lambda_1 g_1 - \lambda_2 g_2)}\right] e^{-a(g_1 - g_2)t} + \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_2} e^{a(\lambda_3 g_3 - \lambda_2 g_2)}\right] e^{a(g_2 - g_3)t}.$$

Hump-shaped due to the two opposing forces: $g_1 > g_2$ pushes labor out of A to M; $g_2 > g_3$ pulls labor out of M to S.

$$s_2'(t) \gtreqless 0 \Leftrightarrow (g_1 - g_2)s_1(t) \gtreqless (g_2 - g_3)s_3(t) \Leftrightarrow g_U(t) = \sum_{k=1}^{3} g_k s_k(t) \gtreqless g_2$$

Initially, $\frac{s_1(t)}{s_3(t)}$ is large; the 1st force is stronger. As $\frac{s_1(t)}{s_3(t)}$ declines over time, the 2nd force becomes stronger eventually.

Characterizing Manufacturing Peak: "^" indicates the peak.

$$s_2'(\hat{t}) = 0 \Leftrightarrow (g_1 - g_2)s_1(\hat{t}) = (g_2 - g_3)s_3(\hat{t}) \Leftrightarrow g_U(\hat{t}) = g_2$$

Peak Time: From $(g_1 - g_2)s_1(\hat{t}) = (g_2 - g_3)s_3(\hat{t})$

$$\hat{t} = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3} + \hat{t}_0, \qquad \text{where } \hat{t}_0 \equiv \frac{1}{a(g_1 - g_3)} \ln\left[\left(\frac{g_1 - g_2}{g_2 - g_3}\right)\frac{\tilde{\beta}_1}{\tilde{\beta}_3}\right]$$

Two Normalizations: Without any loss of generality,

$$\hat{t}_0 = 0 \Leftrightarrow \frac{g_2 - g_3}{g_1 - g_2} = \frac{\tilde{\beta}_1}{\tilde{\beta}_3} \equiv \left[\left(\frac{\beta_1}{\beta_3}\right)^{\frac{1}{1-\sigma}} \frac{\bar{A}_3(0)}{\bar{A}_1(0)}\right]^a$$

The calendar time is reset so that its M-peak would be reached at $\hat{t} = 0$ in the absence of the adoption lags.

$$U(0) = 1 \text{ for } \lambda_1 = \lambda_2 = \lambda_3 = 0 \Leftrightarrow \sum_{k=1}^{3} \tilde{\beta}_k = \sum_{k=1}^{3} \left(\frac{\beta_k^{\frac{1}{1-\sigma}}}{\bar{A}_k(0)}\right)^a = 1.$$

We use the peak time per capita income in the absence of the adoption lags as the *numeraire.*

Note: Under these normalizations, the peak time share of sector-k in the absence of the adoption lags would be $\tilde{\beta}_k$.

Then,

Peak Time

$$\hat{t} = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3}.$$

Peak M-Share

$$\frac{1}{\widehat{s_2}} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{a(g_1 - g_2)\left(\frac{\lambda_1 g_1 - \lambda_2 g_2}{g_1 - g_2} - \hat{t}\right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{a(g_2 - g_3)\left(\hat{t} - \frac{\lambda_2 g_2 - \lambda_3 g_3}{g_2 - g_3}\right)}$$

Peak Time Per Capita Income

$$\widehat{U} = \left\{\sum_{k=1}^{3} \tilde{\beta}_k e^{-a g_k (\hat{t} - \lambda_k)}\right\}^{-\frac{1}{a}}$$

So far, we have looked at the impacts of adoption lags in a single country in isolation, without specifying the sources of the adoption lags.

If we allow countries to differ in $(\lambda_1, \lambda_2, \lambda_3)$, we can perfectly account for $\left(\hat{t}, \widehat{s_2}, \widehat{U}\right)$,

We now restrict ourselves to one-dimension of country heterogeneity, the technology gap, which generate cross-country variations in adoption lags, and study the cross-country implications.

Technology Gaps and Premature Deindustrialization

Consider the world with many countries with

$$(\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3)\lambda$$

$\lambda \geq 0$: **Technology Gap,** Country-specific

$\theta_j > 0$: Sector-specific, capturing the inherent difficulty of technology adoption, common across countries

- **Countries differ only in one dimension**, λ, in their ability to adopt the frontier technologies.
- $\theta_j > 0$ determines how much the technology gap affects the adoption lag in that sector.

$$\frac{A_j(t)}{A_k(t)} = \frac{\bar{A}_j(0)}{\bar{A}_k(0)} e^{-(\theta_j g_j - \theta_k g_k)\lambda} e^{(g_j - g_k)t} \Rightarrow \frac{\partial}{\partial \lambda} \ln\left(\frac{A_j(t)}{A_k(t)}\right) = -(\theta_j g_j - \theta_k g_k)$$

Cross-country productivity difference is larger in sector-j than in sector-k if $\theta_j g_j > \theta_k g_k$.

Proposition 1: Peak Values under the Baumol Effect only

Peak Time:

$$\hat{t}(\lambda) = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3}\lambda.$$

Peak M-Share:

$$\frac{1}{\widehat{s_2}(\lambda)} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{a(g_1 - g_2)\left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2}\lambda - \hat{t}(\lambda)\right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{a(g_2 - g_3)\left(\hat{t}(\lambda) - \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}\lambda\right)}$$

Peak Time Per Capita Income:

$$\widehat{U}(\lambda) = \left\{\sum_{k=1}^{3} \tilde{\beta}_k e^{-a g_k[\hat{t}(\lambda) - \theta_k \lambda]}\right\}^{-\frac{1}{a}}$$

Proposition 2: Conditions for PD with the Baumol (Relative Price) Effect

$$\hat{t}'(\lambda) > 0 \text{ for all } \lambda > 0 \Leftrightarrow \theta_1 g_1 > \theta_3 g_3.$$

With $\theta_1 g_1 > \theta_3 g_3$, the price of A is relatively higher than the price of S in a high-λ country, which delays the peak.

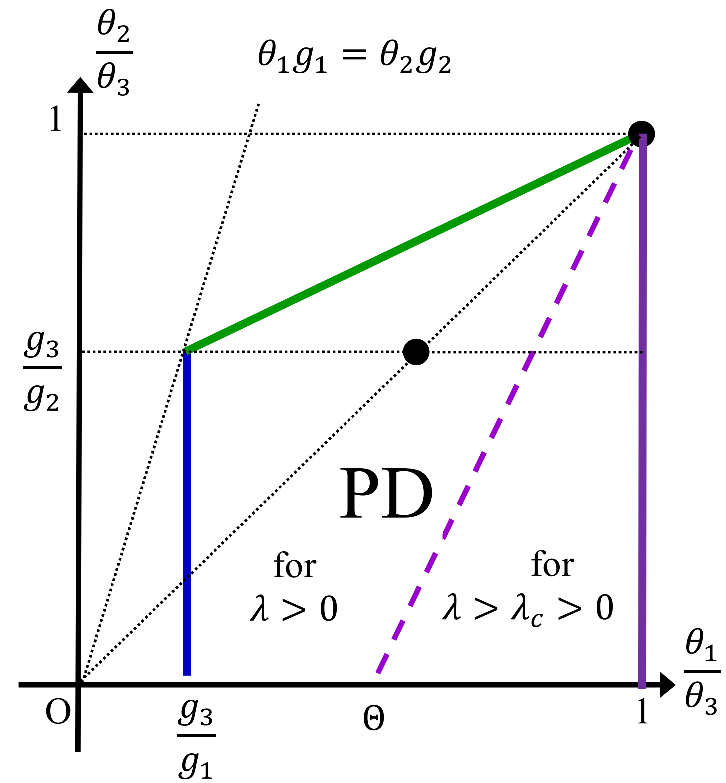
$$\widehat{s_2}'(\lambda) < 0 \text{ for all } \lambda > 0 \Leftrightarrow \left(1 - \frac{g_3}{g_1}\right)\left(\frac{\theta_2}{\theta_3} - 1\right) + \left(1 - \frac{g_3}{g_2}\right)\left(1 - \frac{\theta_1}{\theta_3}\right) < 0$$

With a low θ_2, which has no effect on $\hat{t}$, the price of M is low relative to both A & S in a high-λ country, which keeps the M-share low.

Under the above condition,

$$\widehat{U}'(\lambda) < 0;\ \widehat{U}(\lambda) < \widehat{U}(0) \text{ for } \lambda > \lambda_c \geq 0 \Leftrightarrow \theta_1 < \theta_3 \Leftrightarrow \hat{t}(\lambda) < \theta_1 \lambda < \theta_3 \lambda$$

With $\theta_1 < \theta_3$, the time delay in the peak in a high-λ country is not long enough to make up for the lagging productivity, that is deindustrialization is "premature."



- $\theta_1 g_1 > \max\{\theta_2 g_2, \theta_3 g_3\}$. (productivity differences the largest in A) .
- $\theta_2 g_2 - \theta_3 g_3$ can be either positive or negative.
- $\max\{\theta_1, \theta_2\} < \theta_3$. (adoption lag the longest in S).

Some Examples

Example 1: No Premature Deindustrialization (PD)

Uniform Adoption Lags, as in Krugman (1985)

$$\theta_1 = \theta_2 = \theta_3 = 1 \Leftrightarrow \lambda_1 = \lambda_2 = \lambda_3 = \lambda > 0$$

$$\Longrightarrow \hat{t}(\lambda) = \lambda; \quad \widehat{s_2}(\lambda) = \tilde{\beta}_2; \quad \widehat{U}(\lambda) = 1$$

- The country's technology gap causes a delay in the peak time, $\hat{t}$, by $\lambda > 0$.
- The peak M-share & the peak time per capita income unaffected.

Each country follows the same development path of early industrializers *with a delay*. No PD!!

Thus, the technology gap must have differential impacts on the adoption lags across sectors.

Example 2a-2c: Numerical Illustrations. In all three examples, $\theta_1 = \theta_2 < \theta_3 = 1$ and we use $g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%$; $\alpha = 1/3$, and $\sigma = 0.6$ (hence $a = 6/13$). $\tilde{\beta}_j = 1/3$ for $j = 1,2,3 \Rightarrow \widehat{s_2}(0) = \tilde{\beta}_2 = 1/3$; $\widehat{U}(0) = 1$; $\hat{t}(0) = 0$.

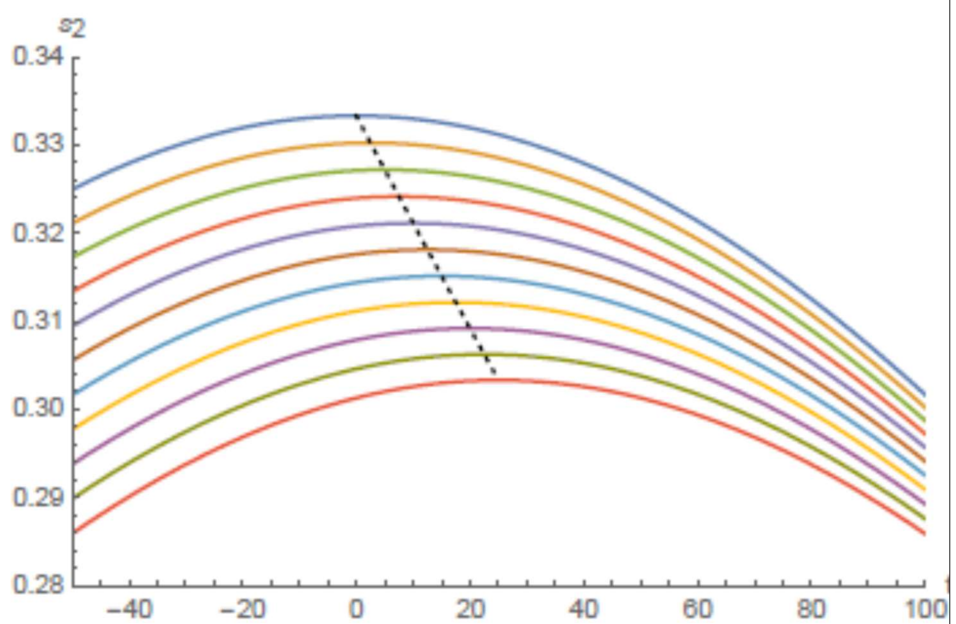
<table>
<tr><th>Example 2a</th><th>$\left(t, s_2(t)\right)$</th><th>$\left(\ln U(t), s_2(t)\right)$</th></tr>
<tr><td>

$$\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.5 = \frac{g_3}{g_2}$$

$$\Longrightarrow \theta_1 g_1 > \theta_2 g_2 = \theta_3 g_3$$

Cross-country productivity differences are the same in M & in S in this case.

</td><td>



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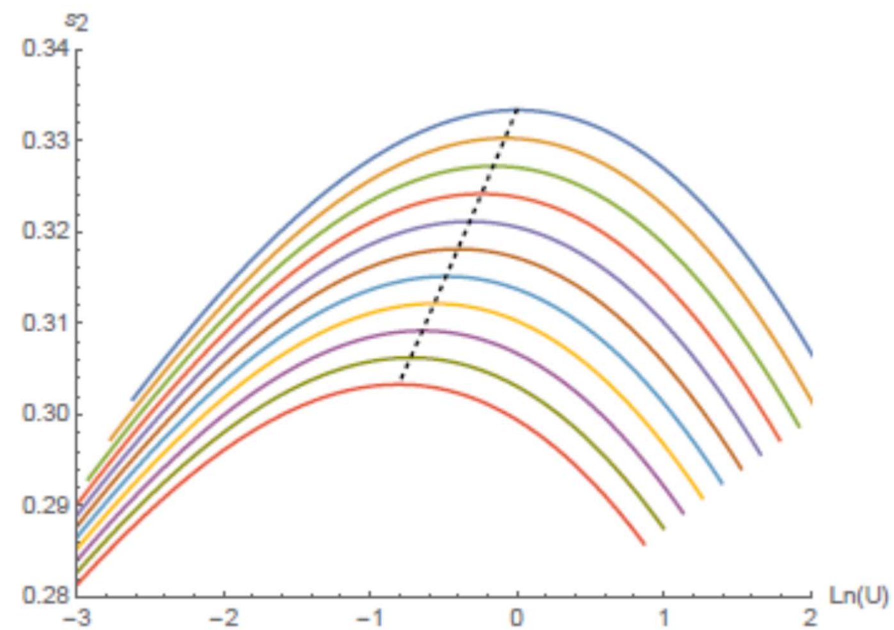

</td></tr>
<tr><td colspan="3">λ=0 λ=10 λ=20 λ=30 λ=40 λ=50 λ=60 λ=70 λ=80 λ=90 λ=100</td></tr>
</table>

<table>
<tr><th></th><th>$\left(t, s_2(t)\right)$</th><th>$\left(\ln U(t)\, , s_2(t)\right)$</th></tr>
<tr><td>

Example 2b

$$\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.35 < \frac{g_3}{g_2}$$
$$\Longrightarrow \theta_1 g_1 > \theta_3 g_3 > \theta_2 g_2$$

Cross-country productivity differences the smallest in M.

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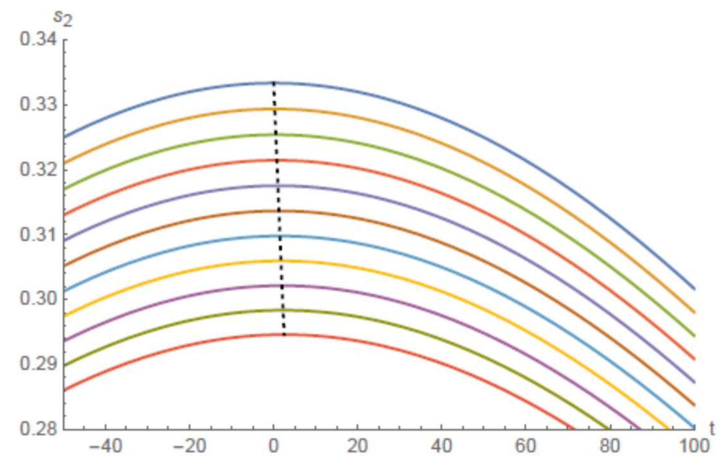

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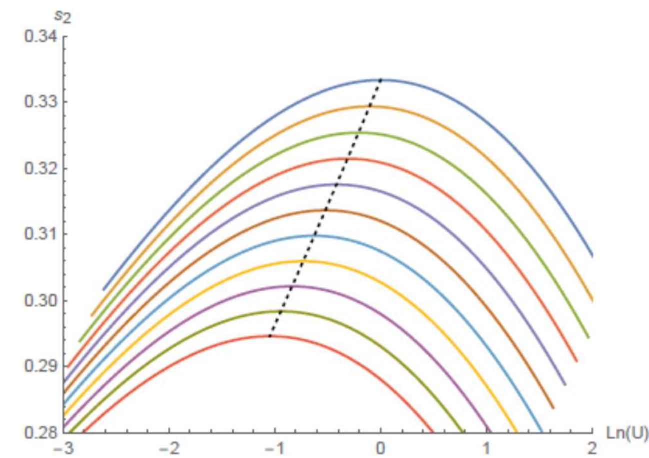

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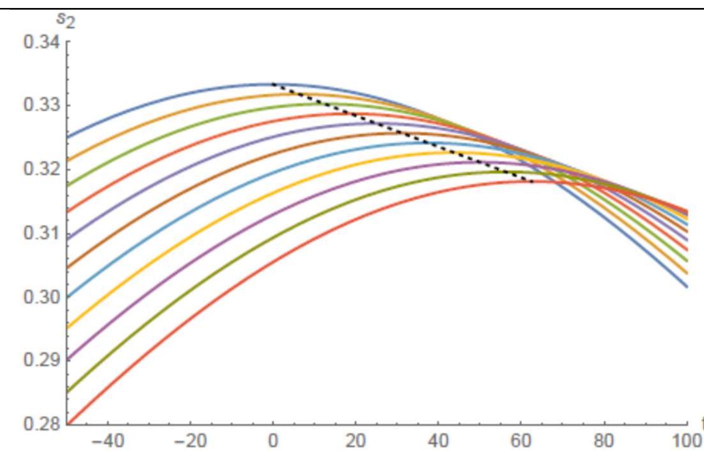
Example 2c

$$\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.75 > \frac{g_3}{g_2}$$
$$\Longrightarrow \theta_1 g_1 > \theta_2 g_2 > \theta_3 g_3$$

Cross-country productivity differences the smallest in S.

</td><td>



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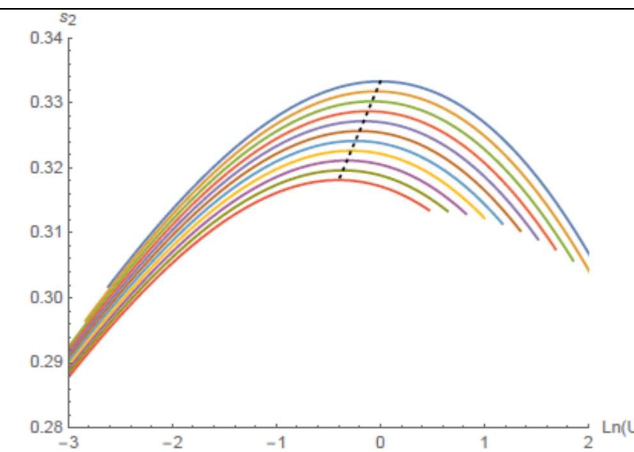


</td></tr>
<tr><td colspan="3">— λ=0 — λ=10 — λ=20 — λ=30 — λ=40 — λ=50 — λ=60 — λ=70 — λ=80 — λ=90 — λ=100</td></tr>
</table>

Some Calibrations:

Rodrik (2016) divided countries into pre-1990 peaked vs. post-1990 peaked.
From his Fig.5, $\hat{t}(\lambda) = 25$ years. From his Table 10,
For the employment shares, $\widehat{s_2}(0) = 21.5\% > \widehat{s_2}(\lambda) = 18.9\%$; $\widehat{U}(\lambda)/\widehat{U}(0) = \widehat{U}(\lambda) = 4273/11048$.
For the value-added shares, $\widehat{s_2}(0) = 27.9\% > \widehat{s_2}(\lambda) = 24.1\%$. $\widehat{U}(\lambda)/\widehat{U}(0) = \widehat{U}(\lambda) = 20537/47099$.

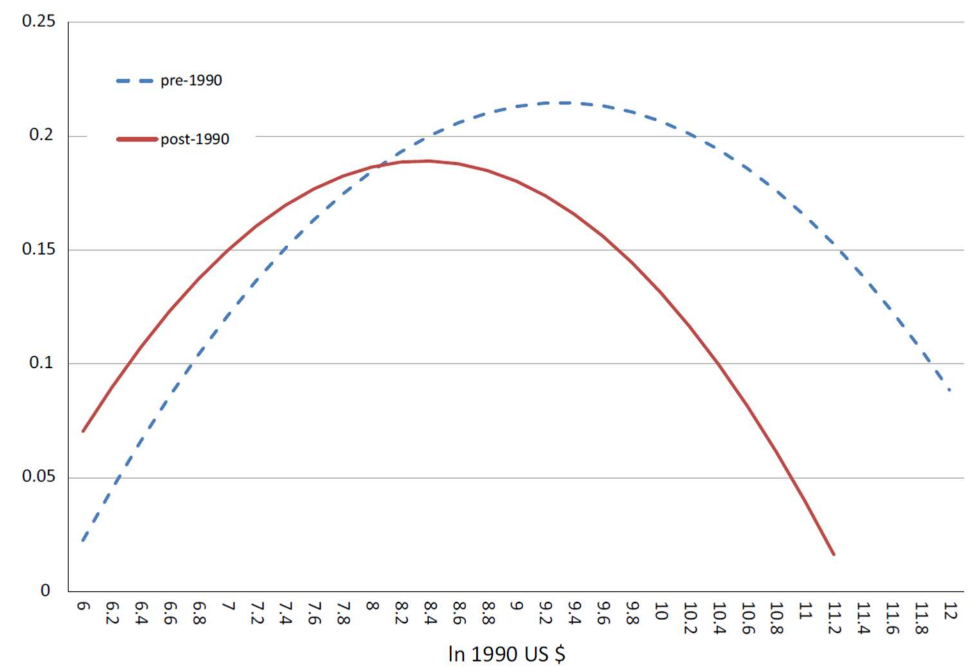


Fig. 6 Simulated manufacturing employment shares

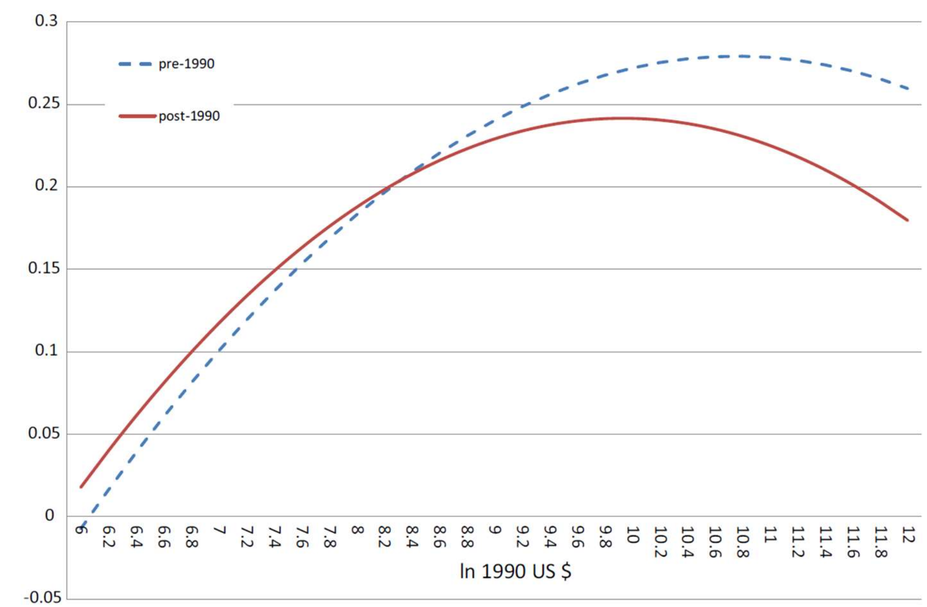


Fig. 7 Simulated manufacturing output shares (MVA/GDP at constant prices)

Peak Time

$$\hat{t}(\lambda) = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3}\lambda.$$

Peak M-Share

$$\frac{1}{\widehat{s_2}(\lambda)} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{a(g_1-g_2)\left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2}\lambda - \hat{t}(\lambda)\right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{a(g_2-g_3)\left(\hat{t}(\lambda) - \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}\lambda\right)}$$

Peak Time Per Capita Income

$$\widehat{U}(\lambda) = \left\{\sum_{k=1}^{3} \tilde{\beta}_k e^{-a g_k [\hat{t}(\lambda) - \theta_k \lambda]}\right\}^{-\frac{1}{a}}$$

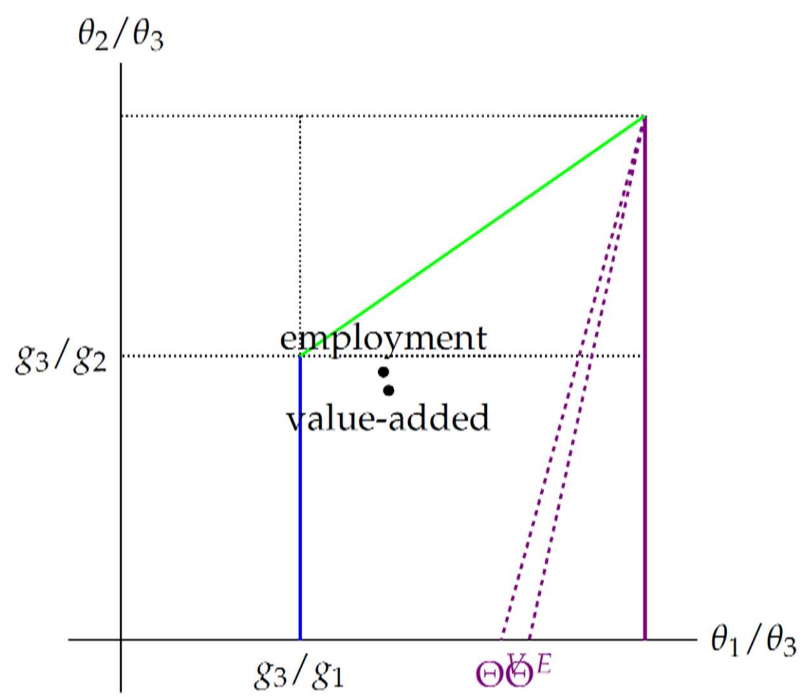
can be inverted into

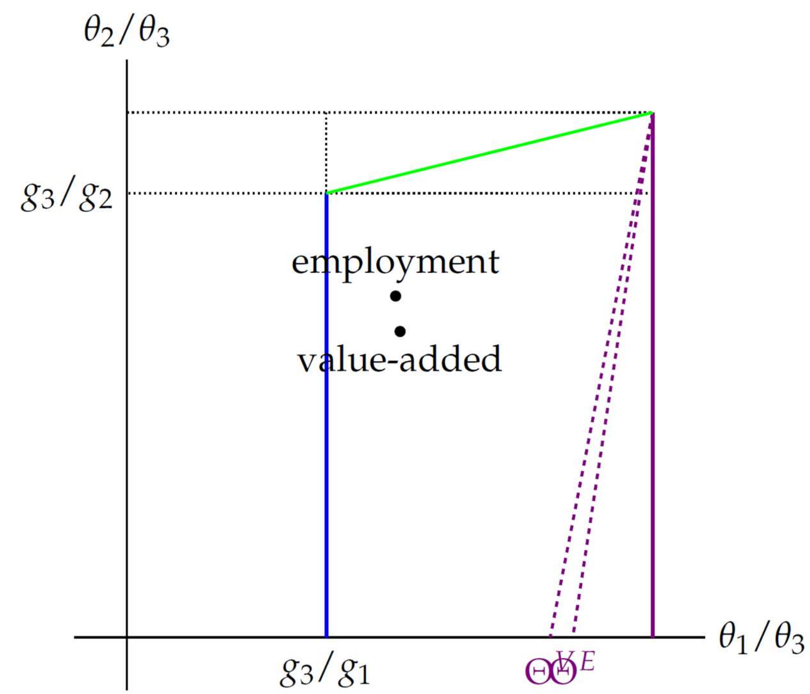
$$\theta_1 \lambda = \hat{t}(\lambda) - \frac{1}{g_1}\ln\left(\widehat{U}(\lambda)\right) + \frac{1}{a g_1}\ln\left(\frac{1-\widehat{s_2}(\lambda)}{1-\widehat{s_2}(0)}\right),$$

$$\theta_2 \lambda = \hat{t}(\lambda) - \frac{1}{g_2}\ln\left(\widehat{U}(\lambda)\right) + \frac{1}{a g_2}\ln\left(\frac{\widehat{s_2}(\lambda)}{\widehat{s_2}(0)}\right).$$

$$\theta_3 \lambda = \hat{t}(\lambda) - \frac{1}{g_3}\ln\left(\widehat{U}(\lambda)\right) + \frac{1}{a g_3}\ln\left(\frac{1-\widehat{s_2}(\lambda)}{1-\widehat{s_2}(0)}\right).$$

	Duarte-Restuccia (2010): $g_1 = 3.8\% > g_2 = 2.4\% > g_3 = 1.3\%$	**Comin et. al. (2021)** $g_1 = 2.9\% > g_2 = 1.3\% > g_3 = 1.1\%$
Empl. Shares	$\left(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}\right) \approx (13.9\%, 28.1\%, 26.0\%)$; $(\theta_1/\theta_3, \theta_2/\theta_3) \approx (0.501, 0.512)$; $\Theta \approx 0.779$.	$\left(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}\right) \approx (17.5\%, 36.9\%, 27.4\%)$ $(\theta_1/\theta_3, \theta_2/\theta_3) \approx (0.511, 0.650)$ and $\Theta \approx 0.848$.
VA Shares	$\left(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}\right) \approx (15.1\%, 32.9\%, 28.2\%)$; $(\theta_1/\theta_3, \theta_2/\theta_3) \approx (0.511, 0.476)$ and $\Theta \approx 0.726$.	$\left(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}\right) \approx (18.9\%, 43.3\%, 29.6\%)$; $(\theta_1/\theta_3, \theta_2/\theta_3) \approx (0.520, 0.583)$ and $\Theta \approx 0.805$





$\theta_1 g_1 > \theta_3 g_3 > \theta_2 g_2 \iff e^{-\theta_1 g_1 \lambda} < e^{-\theta_3 g_3 \lambda} < e^{-\theta_2 g_2 \lambda}$.

Cross-country productivity differences not only the largest in A but also the smallest in M.

1st Extension: Introducing the Engel Effect

The Engel Law through Isoelastic Nonhomothetic CES; Comin-Lashkari-Mestieri (2021), Matsuyama (2019)

$$\left[\sum_{j=1}^{3} (\beta_j)^{\frac{1}{\sigma}} \left(\frac{c_j}{U^{\varepsilon_j}}\right)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \equiv 1$$

Normalize $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3$; with $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$, we go back to the standard homothetic CES.
With $\sigma < 1$, $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3$ $\Rightarrow$ **the income elasticity the lowest in A and the highest in S.**

By maximizing U subject to $\sum_{j=1}^{3} p_j c_j \leq E$,

Expenditure Shares

$$m_j \equiv \frac{p_j c_j}{E} = \frac{\beta_j \left(U^{\varepsilon_j} p_j\right)^{1-\sigma}}{\sum_{k=1}^{3} \beta_k (U^{\varepsilon_k} p_k)^{1-\sigma}} = \beta_j \left(\frac{U^{\varepsilon_j} p_j}{E}\right)^{1-\sigma} \Longrightarrow \frac{m_j}{m_k} = \frac{\beta_j}{\beta_k}\left(\frac{p_j}{p_k} U^{\varepsilon_j - \varepsilon_k}\right)^{1-\sigma}$$

Indirect Utility Function:

$$\left[\sum_{j=1}^{3} \beta_j \left(\frac{U^{\varepsilon_j} p_j}{E}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \equiv 1$$

Cost-of-Living Index:

$$\left[\sum_{j=1}^{3} \beta_j \left(\frac{U^{\varepsilon_j - 1} p_j}{P}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \equiv 1 \Leftrightarrow U \equiv \frac{E}{P}$$

Income Elasticity:

$$\eta_j \equiv \frac{\partial \ln c_j}{\partial \ln(U)} = 1 + \frac{\partial \ln m_j}{\partial \ln(E/P)} = 1 + (1-\sigma)\left\{\varepsilon_j - \sum_{k=1}^{3} m_k \varepsilon_k\right\}$$

Structural Change with the Engel (Income) Effect: Let $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 = 3 - \varepsilon_1 - \varepsilon_2$.
Then, *even with constant relative prices,*

Decline of Agriculture: $s_1(t) = m_1(t)$ is decreasing in $U(t)$, because

$$\frac{1}{s_1(t)} - 1 = \frac{m_2(t)}{m_1(t)} + \frac{m_3(t)}{m_1(t)} = \frac{\beta_2}{\beta_1}\left(\frac{p_2}{p_1}U(t)^{\varepsilon_2-\varepsilon_1}\right)^{1-\sigma} + \frac{\beta_3}{\beta_1}\left(\frac{p_3}{p_1}U(t)^{\varepsilon_3-\varepsilon_1}\right)^{1-\sigma}$$

Rise of Services: $s_3(t) = m_3(t)$ is increasing in $U(t)$, because

$$\frac{1}{s_3(t)} - 1 = \frac{m_1(t)}{m_3(t)} + \frac{m_2(t)}{m_3(t)} = \frac{\beta_1}{\beta_3}\left(\frac{p_1}{p_3}U(t)^{\varepsilon_1-\varepsilon_3}\right)^{1-\sigma} + \frac{\beta_2}{\beta_3}\left(\frac{p_2}{p_3}U(t)^{\varepsilon_2-\varepsilon_3}\right)^{1-\sigma}$$

Rise and Fall of Manufacturing: $s_2(t) = m_2(t)$ is hump-shaped in $U(t)$, because

$$\frac{1}{s_2(t)} - 1 = \frac{m_1(t)}{m_2(t)} + \frac{m_3(t)}{m_2(t)} = \frac{\beta_1}{\beta_2}\left(\frac{p_1}{p_2}U(t)^{\varepsilon_1-\varepsilon_2}\right)^{1-\sigma} + \frac{\beta_3}{\beta_2}\left(\frac{p_3}{p_2}U(t)^{\varepsilon_3-\varepsilon_2}\right)^{1-\sigma}.$$

Hump-shaped due to the two opposing forces: $\varepsilon_1 < \varepsilon_2$ pushes labor out of A to M; $\varepsilon_2 < \varepsilon_3$ pulls labor out of M to S.

$$s_2'(t) = m_2'(t) \gtreqless 0 \Leftrightarrow (\varepsilon_2 - \varepsilon_1)\frac{m_1(t)}{m_2(t)} \gtreqless (\varepsilon_3 - \varepsilon_2)\frac{m_3(t)}{m_2(t)} \Leftrightarrow \eta_2 \gtreqless 1$$

Initially, when A is large & S is small, the former effect is stronger. Over time, A shrinks & S expands, and eventually, the latter effect becomes stronger.

The production side is the same as before. By following the same step, we obtain

Equilibrium Shares

$$s_j = \frac{\left[\beta_j^{\frac{1}{\sigma-1}} A_j\right]^{-a}}{[U^{\varepsilon_j}]^{-a}}, \qquad \text{where } \sum_{k=1}^{3} \frac{\left[\beta_k^{\frac{1}{\sigma-1}} A_k\right]^{-a}}{[U^{\varepsilon_k}]^{-a}} \equiv 1$$

With $A_j(t) = \bar{A}_j(t-\lambda_j) = \bar{A}_j(0)e^{g_j(t-\theta_j\lambda)}$,

$s_2(t)$:

$$\frac{1}{s_2(t)} = U(t)^{a(\varepsilon_1-\varepsilon_2)}\left[\frac{\tilde{\beta}_1}{\tilde{\beta}_2} e^{a(\theta_1 g_1-\theta_2 g_2)\lambda}\right] e^{-a(g_1-g_2)t} + 1 + U(t)^{a(\varepsilon_3-\varepsilon_2)}\left[\frac{\tilde{\beta}_3}{\tilde{\beta}_2} e^{a(\theta_3 g_3-\theta_2 g_2)\lambda}\right] e^{a(g_2-g_3)t}$$

$U(t)$:

$$U(t)^{a\varepsilon_1}\tilde{\beta}_1 e^{-ag_1(t-\theta_1\lambda)} + U(t)^{a\varepsilon_2}\tilde{\beta}_2 e^{-ag_2(t-\theta_2\lambda)} + U(t)^{a\varepsilon_3}\tilde{\beta}_3 e^{-ag_3(t-\theta_3\lambda)} \equiv 1$$

$s_2'(t) = 0$:

$$(g_1-g_2) = (g_2-g_3)U^{a(\varepsilon_3-\varepsilon_2)}\left[\frac{\tilde{\beta}_3}{\tilde{\beta}_1}\right] e^{a(\theta_3 g_3-\theta_1 g_1)\lambda} e^{a(g_1-g_3)t}$$
$$+ \frac{\left\{(\varepsilon_1-\varepsilon_2) + (\varepsilon_3-\varepsilon_2)U^{a(\varepsilon_3-\varepsilon_1)}\left[\frac{\tilde{\beta}_3}{\tilde{\beta}_1}\right] e^{a(\theta_3 g_3-\theta_1 g_1)\lambda} e^{a(g_1-g_3)t}\right\}\left\{g_1 U^{a(\varepsilon_1-\varepsilon_2)}\tilde{\beta}_1 e^{-ag_1(t-\theta_1\lambda)} + g_2\tilde{\beta}_2 e^{-ag_2(t-\theta_2\lambda)} + g_3 U^{a(\varepsilon_3-\varepsilon_2)}\tilde{\beta}_3 e^{-ag_3(t-\theta_3\lambda)}\right\}}{\varepsilon_1 U^{a(\varepsilon_1-\varepsilon_2)}\tilde{\beta}_1 e^{-ag_1(t-\theta_1\lambda)} + \varepsilon_2\tilde{\beta}_2 e^{-ag_2(t-\theta_2\lambda)} + \varepsilon_3 U^{a(\varepsilon_3-\varepsilon_2)}\tilde{\beta}_3 e^{-ag_3(t-\theta_3\lambda)}}.$$

$\hat{t}$ and $\hat{U}$ solve the equation for $U(t)$ and the equation for $s_2'(t) = 0$, simultaneously.
Then, $\hat{s}_2$ can be obtained by plugging $\hat{t}$ and $\hat{U}$ into the equation for $s_2(t)$

(Analytically Solvable) Case: $0 < \mu \equiv \frac{\varepsilon_2 - \varepsilon_1}{g_1 - g_2} = \frac{\varepsilon_3 - \varepsilon_2}{g_2 - g_3} < \frac{1}{g_1 - \bar{g}}$, where $\bar{g} \equiv \frac{g_1 + g_2 + g_3}{3}$

Proposition 3 (Impact of Adding the Engel Effect on top of the Baumol Effect)

Peak Time

$$\hat{t}(\lambda;\mu) = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3}\lambda - \mu \ln \widehat{U}(\lambda;\mu) = \hat{t}(\lambda;0) - \frac{\mu}{1+\mu\bar{g}} \ln \widehat{U}(\lambda;0)$$

Peak M-Share

$$\frac{1}{\widehat{s_2}(\lambda;\mu)} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{a(g_1-g_2)\left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2}\lambda - \hat{t}(\lambda;0)\right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{a(g_2-g_3)\left(\hat{t}(\lambda;0) - \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}\lambda\right)} = \frac{1}{\widehat{s_2}(\lambda;0)}$$

Peak Time Per Capita Income

$$\widehat{U}(\lambda;\mu) = \left\{\sum_{k=1}^{3} \tilde{\beta}_k e^{-a g_k[\hat{t}(\lambda;0) - \theta_k \lambda]}\right\}^{-\frac{1}{a}\left(\frac{1}{1+\mu\bar{g}}\right)} = \widehat{U}(\lambda;0)^{\left(\frac{1}{1+\mu\bar{g}}\right)}$$

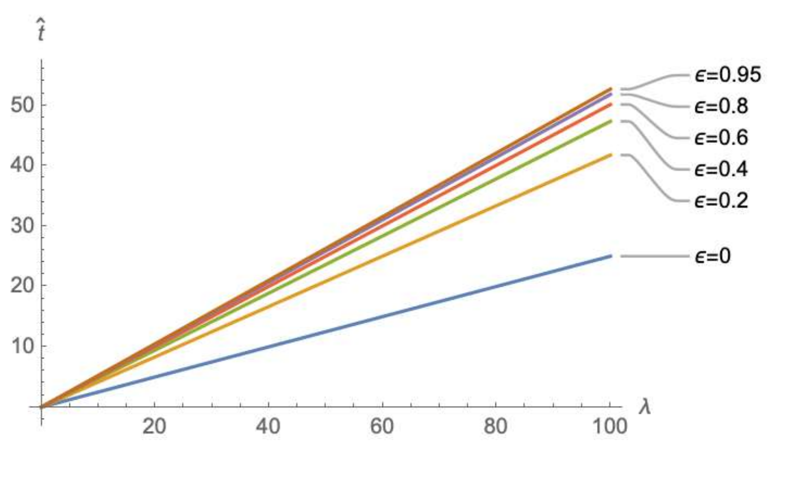
- $\widehat{s_2}'(\lambda;\mu) < 0$; $\widehat{U}'(\lambda;\mu) < 0$ under the same condition; $\hat{t}'(\lambda;\mu) > 0$ under a weaker condition.
- Fixing g_1, g_2, g_3, a higher μ has
  - **No effect** on the peak values of the frontier country, $\hat{t}(0;\mu), \widehat{s_2}(0;\mu), \widehat{U}(0;\mu)$.
  - **A further delay** in $\hat{t}(\lambda;\mu)$ for every country with $\lambda > 0$.
  - **No effect** on $\widehat{s_2}(\lambda;\mu)$ for every country with $\lambda > 0$.
  - **A smaller decline** in $\widehat{U}(\lambda;\mu)$ for each country with $\lambda > 0$.

Analytically Solvable Case: A Numerical Illustration

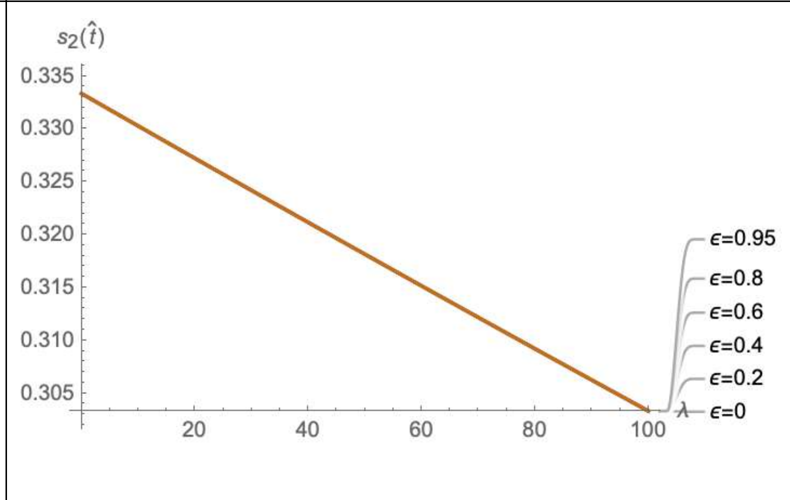
$g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%$, $\theta = 0.5$, $a = 6/13$; $\tilde{\beta}_j = 1/3$ for $j = 1,2,3$.

In this case, $g_1 - g_2 = g_2 - g_3 = \bar{g} = 1.2\% > 0 \Longrightarrow \varepsilon_1 = 1 - \epsilon < \varepsilon_2 = 1 < \varepsilon_3 = 1 + \epsilon$ for $0 < \epsilon = (1.2\%)\mu < 1$

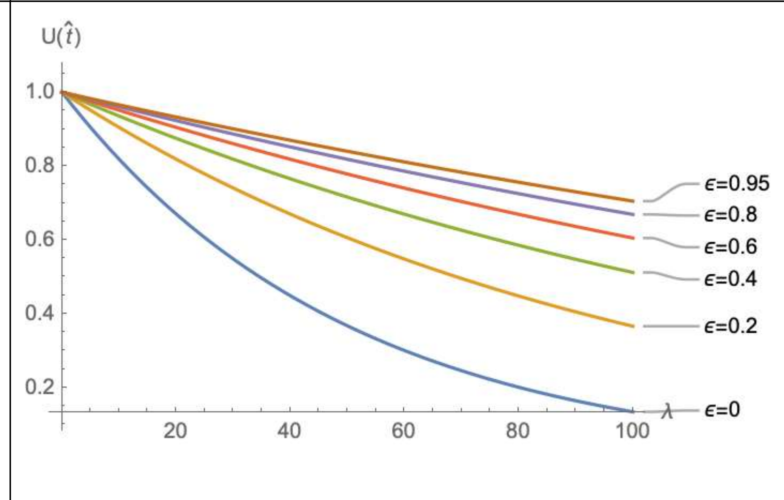
Peak Time



Peak M-Share

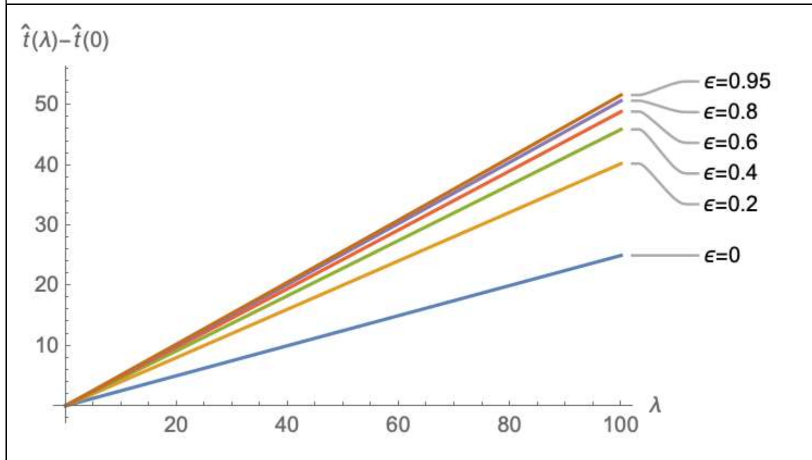
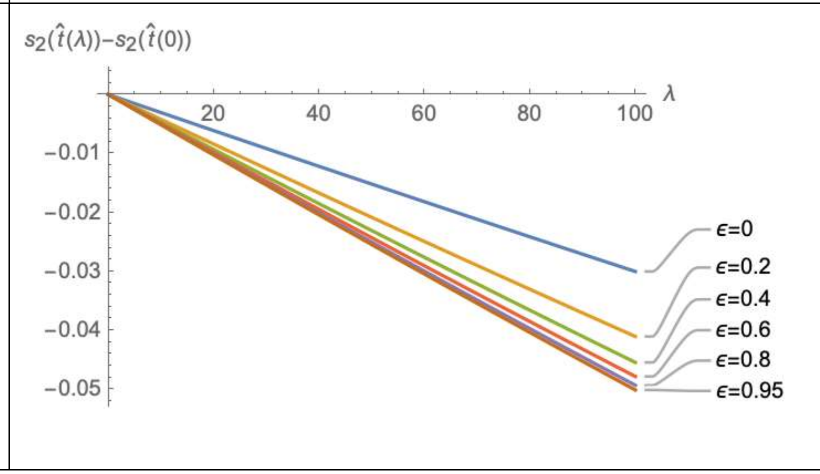
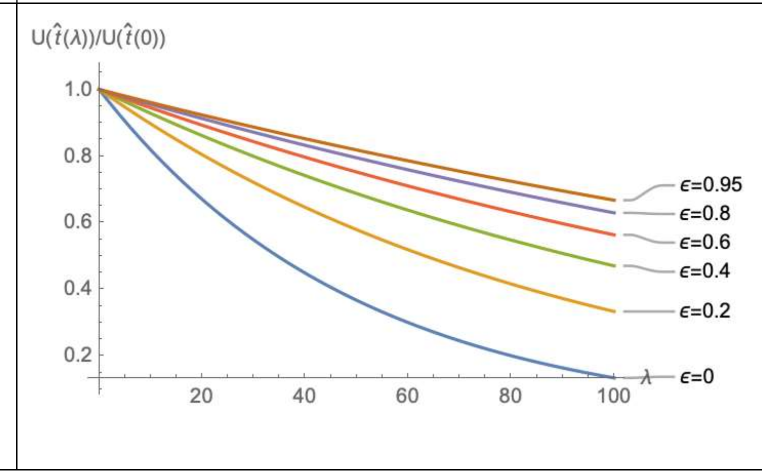


Peak Time Per Capita Income



(Empirically More Plausible) Case:

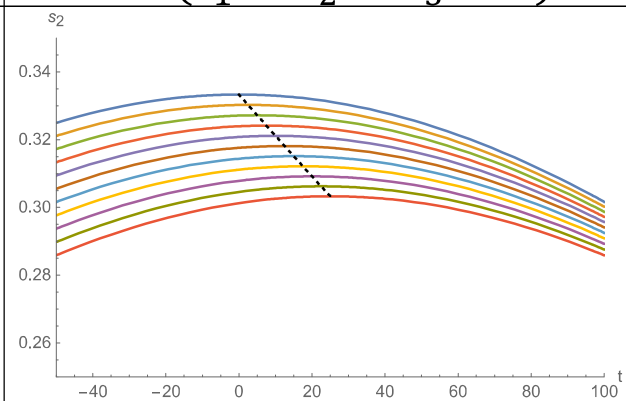
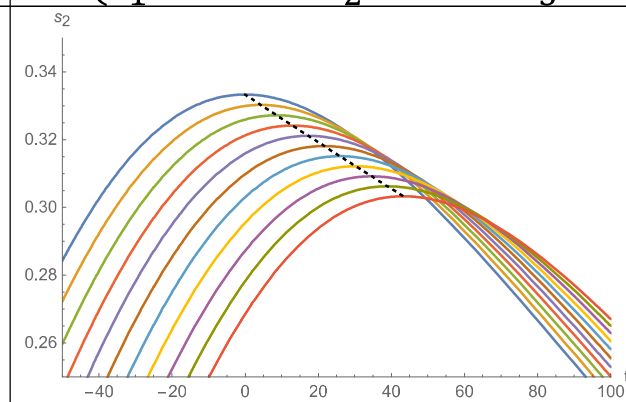
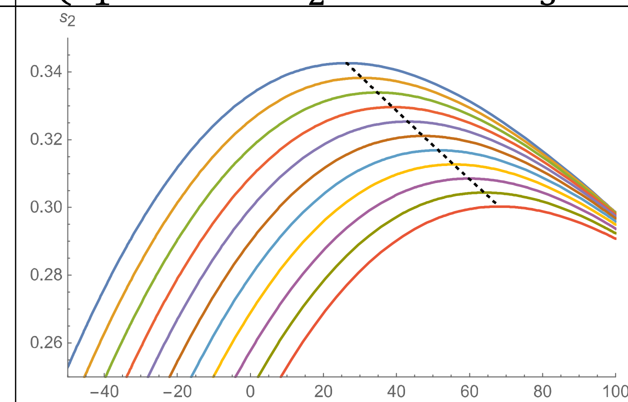
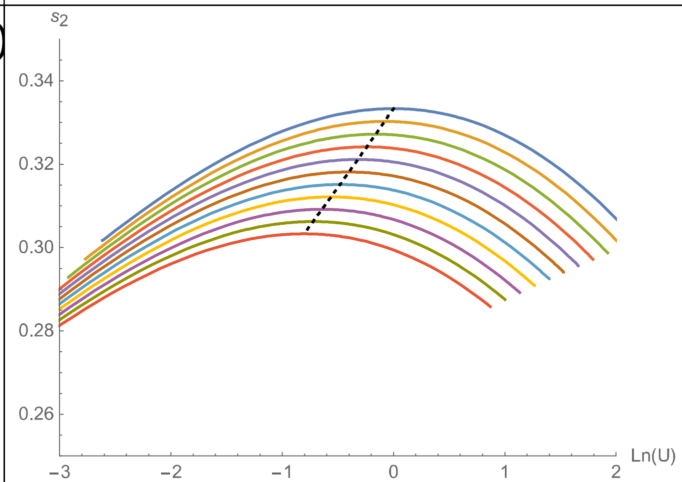
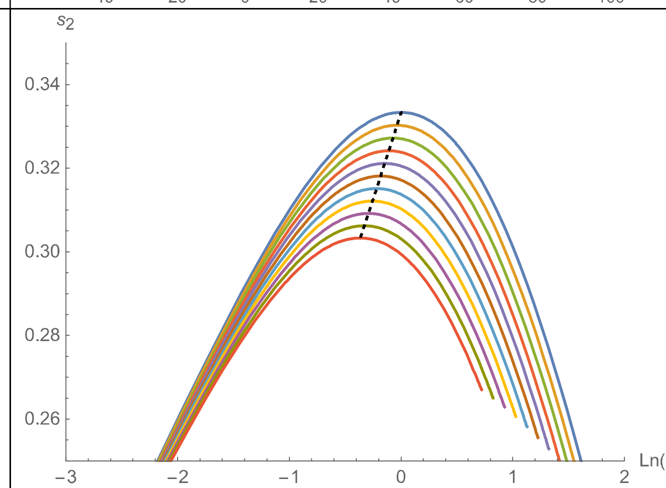
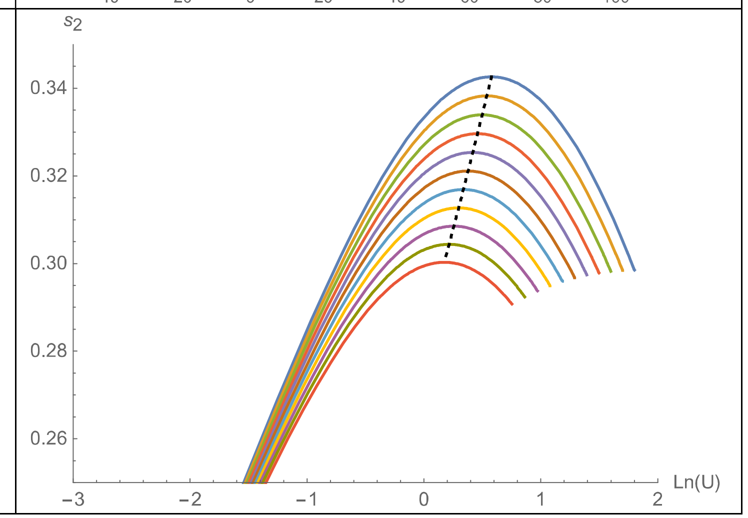
$$\varepsilon_1 = 1 - \epsilon < \varepsilon_2 = 1 + \frac{\epsilon}{3} < \varepsilon_3 = 1 + \frac{2\epsilon}{3} \text{ for } 0 < \epsilon < 1 \Longrightarrow \frac{g_1 - g_2}{g_2 - g_3} = 1 < \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_3 - \varepsilon_2} = 4, \text{ as in CLM (2021).}$$

Peak Time	**Peak M-Share**	**Peak Time Per Capita Income**
		

In this case, the frontier country's peak values are affected by ϵ. Relative to the frontier country, a higher ϵ causes a high-λ country to have

- A further delay in $\hat{t}(\lambda;\mu)$
- A *larger* decline in $\widehat{s_2}(\lambda;\mu)$
- A smaller decline in $\widehat{U}(\lambda;\mu)$.

Stronger nonhomotheticity changes the shape of the time paths significantly.
It does not change the implications on PD, i.e., how technology gaps affect $\hat{t}, s_2(\hat{t})$, and $U(\hat{t})$.

	$\epsilon = 0$	$\epsilon = 0.6$	
	$(\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1)$	$(\varepsilon_1 = .4 < \varepsilon_2 = 1 < \varepsilon_3 = 1.6)$	$(\varepsilon_1 = .4 < \varepsilon_2 = 1.2 < \varepsilon_3 = 1.4)$
$(t, s_2(t))$			
$(\ln U(t), s_2(t))$			

What happens if we had *solely* the Engel effect with $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 = 3 - \varepsilon_1 - \varepsilon_2$, without the Baumol effect, $g_1 = g_2 = g_3 = \bar{g} > 0$?

Under the two normalizations

$$\left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_3 - \varepsilon_2}\right)\frac{\tilde{\beta}_1}{\tilde{\beta}_3} = 1; \;\; \tilde{\beta}_1 + \tilde{\beta}_2 + \tilde{\beta}_3 = 1$$

which ensures $\widehat{U}(0) = 1$ and $\hat{t}(0) = 0$,

Proposition 4: Peak Values under the Engel (Income) Effect only

Peak Time

$$\hat{t}(\lambda) = \frac{1}{a\bar{g}}\ln\left\{\sum_{k=1}^{3}\tilde{\beta}_k e^{a(\theta_k\bar{g}\lambda + \varepsilon_k \ln \widehat{U}(\lambda))}\right\}$$

Peak M-Share

$$\frac{1}{\widehat{s_2}(\lambda)} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right)e^{a(\varepsilon_2-\varepsilon_1)\left(-\frac{\theta_2-\theta_1}{\varepsilon_2-\varepsilon_1}\bar{g}\lambda - \ln \widehat{U}(\lambda)\right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right)e^{a(\varepsilon_3-\varepsilon_2)\left(\ln \widehat{U}(\lambda) - \frac{\theta_2-\theta_3}{\varepsilon_3-\varepsilon_2}\bar{g}\lambda\right)}$$

Peak Time Per Capita Income

$$\ln \widehat{U}(\lambda) = \frac{\theta_1 - \theta_3}{\varepsilon_3 - \varepsilon_1}\bar{g}\lambda$$

Proposition 5: Conditions for PD with the Engel (Income) Effect Only

$$\hat{U}'(\lambda) < 0 \text{ for all } \lambda > 0 \Leftrightarrow 0 < \frac{\theta_1}{\theta_3} < 1$$

With a low θ_1 and a high θ_3, the price of the income elastic S is high relative to the income inelastic A in a high-λ country, which make it necessary to reallocate labor to S at earlier stage of development.

$$\hat{s_2}'(\lambda) < 0 \text{ for all } \lambda > 0 \Leftrightarrow \left(1 - \frac{\varepsilon_2}{\varepsilon_3}\right)\left(\frac{\theta_1}{\theta_3} - 1\right) + \left(1 - \frac{\varepsilon_1}{\varepsilon_3}\right)\left(1 - \frac{\theta_2}{\theta_3}\right) > 0.$$

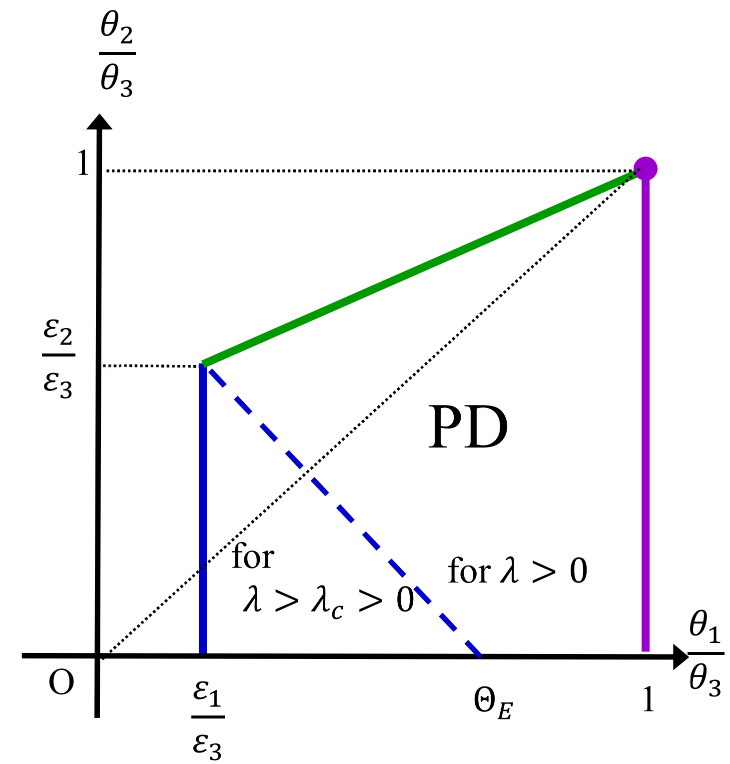
With a low θ_2, which has no effect on $\hat{U}(\lambda)$, the price of M is low relative to both A & S in a high-λ country, which keeps the M-share low.

Under the above condition,

$$\hat{t}'(\lambda) > 0 \text{ for a sufficiently large } \lambda \Leftrightarrow \frac{\theta_1}{\theta_3} > \frac{\varepsilon_1}{\varepsilon_3}$$

$$\hat{t}'(\lambda) > 0 \text{ for all } \lambda > 0 \Leftrightarrow \left(\Theta_E - \frac{\varepsilon_1}{\varepsilon_3}\right)\left[1 - \left(\frac{\varepsilon_3}{\varepsilon_2}\right)\frac{\theta_2}{\theta_3}\right] < \frac{\theta_1}{\theta_3} - \frac{\varepsilon_1}{\varepsilon_3} < 1 - \frac{\varepsilon_1}{\varepsilon_3}$$

where $\varepsilon_1/\varepsilon_3 < \Theta_E < 1$.



With $g_1 = g_2 = g_3 = \bar{g}$, PD occurs only if $\theta_1\bar{g}, \theta_2\bar{g} < \theta_3\bar{g}$, that is, when cross-country productivity difference is *the largest in S.*

2nd Extension: Introducing International Trade

One Implication of PD (consistent with the empirical evidence):

$$\frac{\partial}{\partial\lambda}\ln\left(\frac{A_1(t)}{A_2(t)}\right) = -(\theta_1 g_1 - \theta_2 g_2) < 0.$$

- A low-λ country has comparative advantage in A and a high-λ country has comparative advantage in M.
- Opening up trade in A and in M would weaken PD by allowing high-λ country to export M.
- Consistent with the findings that East Asia "suffers" less from PD.

A Two-Country Technology Gap Model of PD: $\lambda^1 < \lambda^2$ (Superscript indicates the country)

Trade Cost: Only $e^{-\tau_1} < 1$ fraction of A and only $e^{-\tau_2} < 1$ fraction of M shipped arrive to the destination.

$$m_j^c = \beta_j\left(\frac{p_j^c}{P^c}\right)^{1-\sigma};\quad P^c = \left[\sum_{k=1}^{3}\beta_k (p_k^c)^{1-\sigma}\right]^{1/(1-\sigma)} \qquad \& \qquad s_j^c = \left(A_j^c\right)^{\frac{1}{\alpha}}\left(\frac{p_j^c}{E^c}\right)^{\frac{1}{\alpha}};\ E^c = \left[\sum_{k=1}^{3}(A_k^c)^{\frac{1}{\alpha}}(p_k^c)^{\frac{1}{\alpha}}\right]^{\alpha}$$

With $g_1\theta_1 > g_2\theta_2$, Leader (Country-1) has CA in A and Laggard (Country-2) has CA in M.

1 may export A to 2: $e^{\tau_1}p_1^1 \geq p_1^2;\ e^{-\tau_1}[A_1^1(s_1^1)^{1-\alpha} - c_1^1]L^1 = [c_1^2 - A_1^2(s_1^2)^{1-\alpha}]L^2 \geq 0.$ $\rightarrow$ $[s_1^1 - m_1^1]E^1L^1 = [m_1^2 - s_1^2]E^2L^2 \geq 0.$

2 may export M to 1: $p_2^1 \leq e^{\tau_2}p_2^2; [c_2^1 - A_2^1(s_2^1)^{1-\alpha}]L^1 = e^{-\tau_2}[A_2^2(s_2^2)^{1-\alpha} - c_2^2]L^2 \geq 0.$ $\rightarrow$ $[m_2^1 - s_2^1]E^1L^1 = [s_2^2 - m_2^2]E^2L^2 \geq 0.$

S is nontradeable: $p_3^1 \neq p_3^2;\ c_3^1 = A_3^1(s_3^1)^{1-\alpha};\ c_3^2 = A_3^2(s_3^2)^{1-\alpha}$ $\rightarrow$ $m_3^1 = s_3^1;\ m_3^2 = s_3^2.$

Condition for No Trade Equilibrium:

$$e^{\tau_1+\tau_2} > \frac{p_2^1(t)}{p_1^1(t)}\frac{p_1^2(t)}{p_2^2(t)} = \left[\frac{A_2^1(t)}{A_1^1(t)}\frac{A_1^2(t)}{A_2^2(t)}\right]^{-\frac{a}{(1-\sigma)}} = e^{\frac{a(g_1\theta_1-g_2\theta_2)}{(1-\sigma)}(\lambda^2-\lambda^1)}$$

$$\Leftrightarrow \tau_1+\tau_2 > T_+ \equiv \frac{a(g_1\theta_1-g_2\theta_2)}{(1-\sigma)}(\lambda^2-\lambda^1) > 0.$$

Trade Equilibrium under

$$0 < \tau_1+\tau_2 \leq T_+ \equiv \frac{a(g_1\theta_1-g_2\theta_2)}{(1-\sigma)}(\lambda^2-\lambda^1).$$

Then, 1 exports A to 2 and imports M from 2.

Equilibrium Conditions:

$$s_1^1+s_2^1 = m_1^1+m_2^1;\ s_1^2+s_2^2 = m_1^2+m_2^2$$

$$[s_1^1-m_1^1]E^1L^1 = [s_2^2-m_2^2]E^2L^2 > 0$$

$$e^{\tau_1}p_1^1 = p_1^2;\ p_2^1 = e^{\tau_2}p_2^2$$

Impact of International Trade (Numerical Simulation): $L_1/L_2 = 1$.

$$0 < \tau \equiv \frac{\tau_1 + \tau_2}{T_+} \equiv \frac{(1-\sigma)(\tau_1 + \tau_2)}{a(g_1\theta_1 - g_2\theta_2)(\lambda^2 - \lambda^1)} < 1$$

$$\Longrightarrow \ 1 < \frac{p_2^1}{p_1^1}\frac{p_1^2}{p_2^2} = e^{\tau_1+\tau_2} = e^{\tau T_+} \leq e^{T_+}.$$

We plot how the peak values change in response to τ.

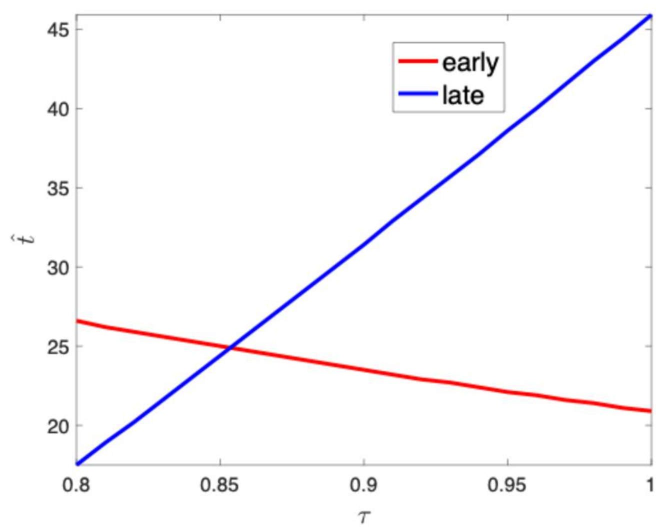
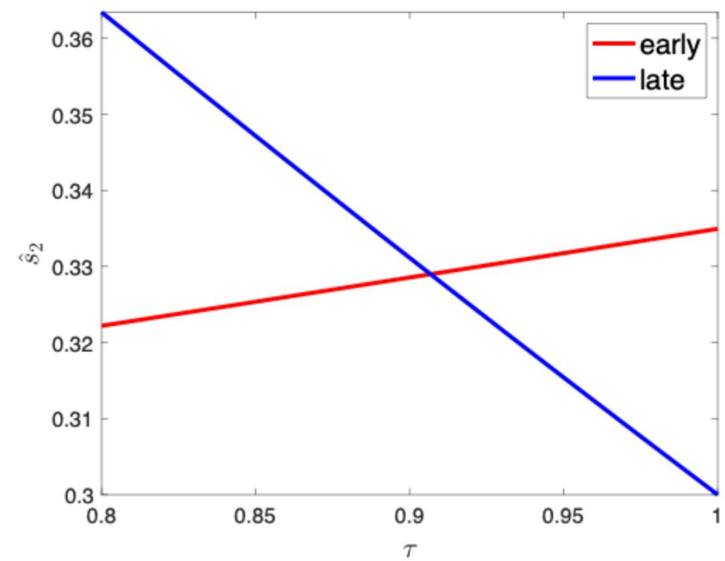
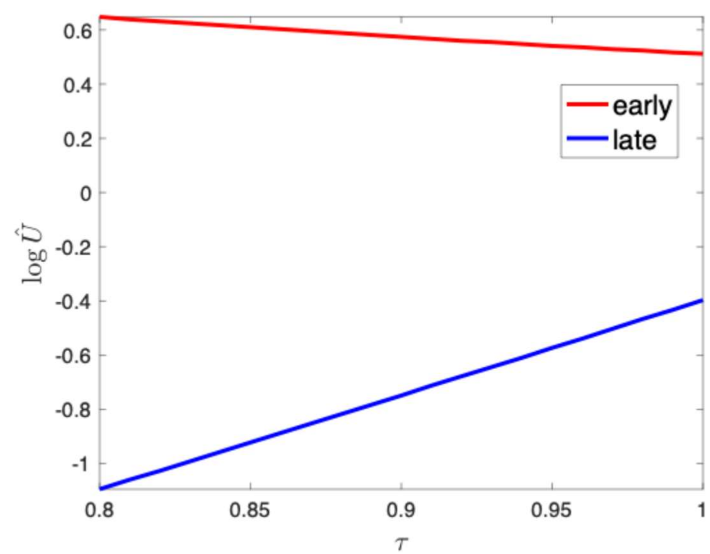
In all four cases, our mechanism for PD is:

- Robust to introducing international trade.
- Weaker in that the differences btw the leader and the laggard in $\hat{t}$ and $\hat{s}_2$ become smaller (larger in $\widehat{U}$ in $\widehat{m}_2$), as τ declines. For a sufficiently small τ, the reversal occurs in $\hat{t}$ and $\hat{s}_2$.

PD holds, when the trade cost accounts for more than about 1/3 of the imported goods prices, empirically plausible.

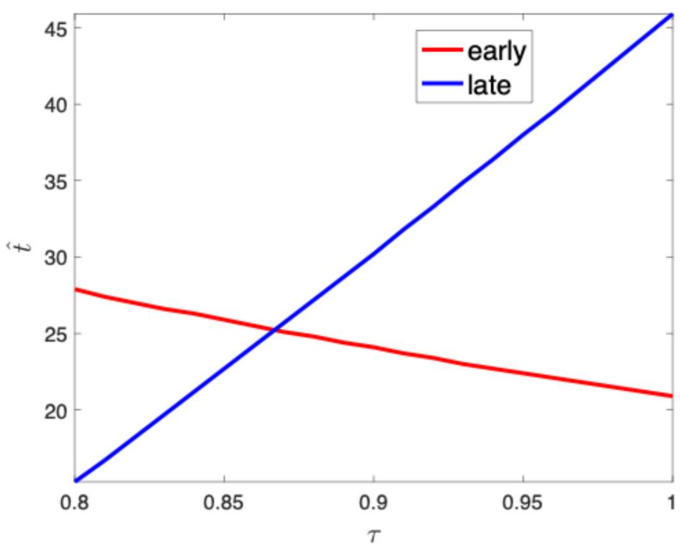
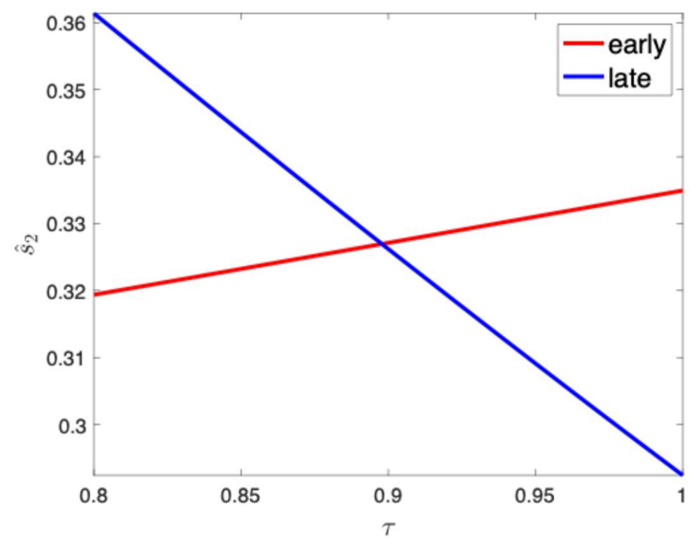
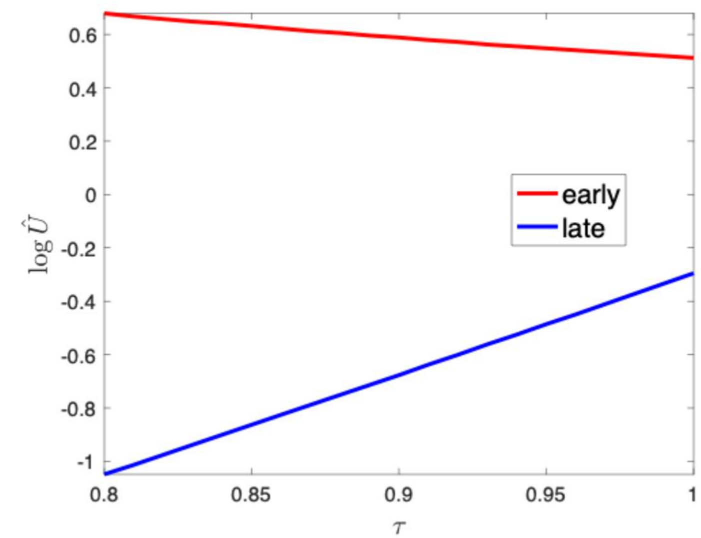
Under our mechanism, PD occurs not because of international trad but in spite of international trade.

Duarte-Restuccia productivity growth rates;
Employment Shares

Peak Time	Peak M-Share	Peak Time Per Capita Income
		

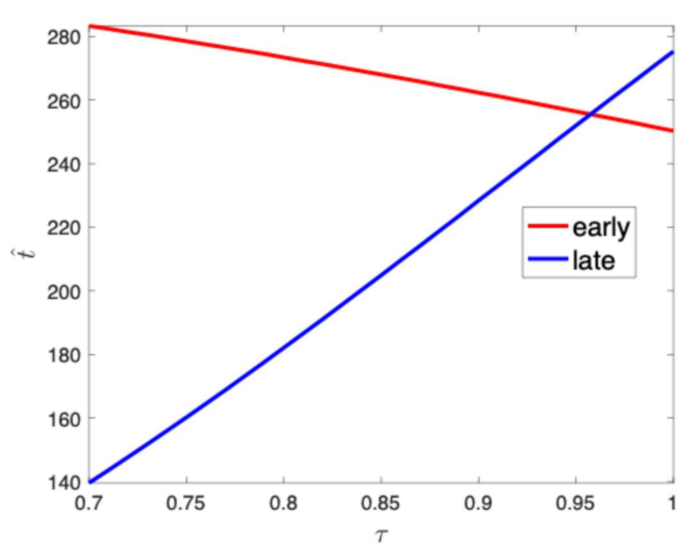
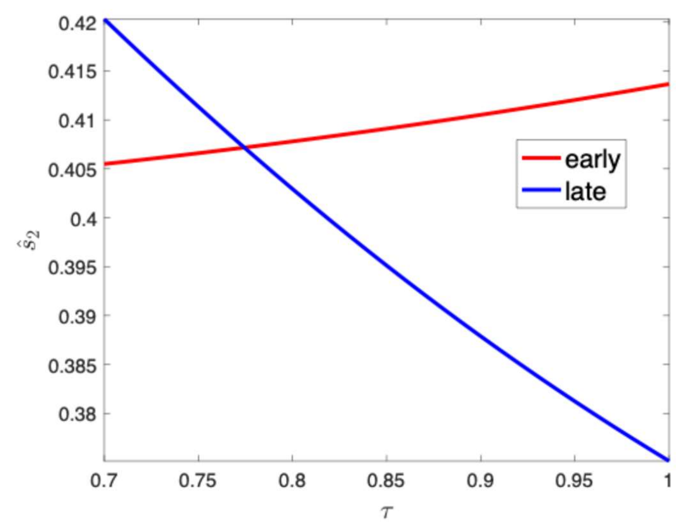
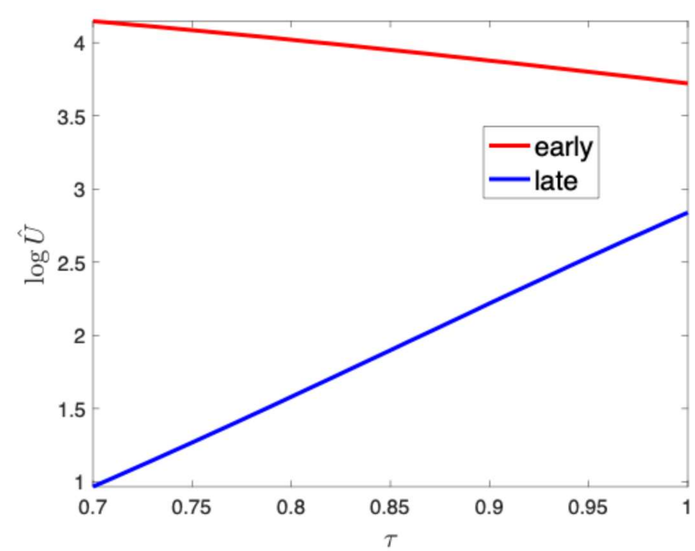
Reversal of $\hat{t}$ at $\tau \approx 0.85$, or $e^{\tau_1+\tau_2} = e^{\tau T_+} \approx 1.986 \rightarrow \sqrt{1.986} \approx 1.41$ times higher in the importing country.
Reversal of $\hat{s}_2$ at $\tau \approx 0.91$ or $e^{\tau_1+\tau_2} = e^{\tau T_+} \approx 2.242 \rightarrow \sqrt{2.242} \approx 1.497$ times higher in the importing country.

Duarte-Restuccia productivity growth rates;
Value-Added Shares

Peak Time	Peak M-Share	Peak Time Per Capita Income
		

Reversal of $\hat{t}$ at $\tau \approx 0.87$ or $e^{\tau_1+\tau_2} \approx 2.185 \rightarrow \sqrt{2.185} \approx 1.478$ times higher in the importing country.
Reversal of $\hat{s}_2$ at $\tau \approx 0.90$ or $e^{\tau_1+\tau_2} \approx 2.244 \rightarrow \sqrt{2.244} \approx 1.498$ times higher in the importing country.

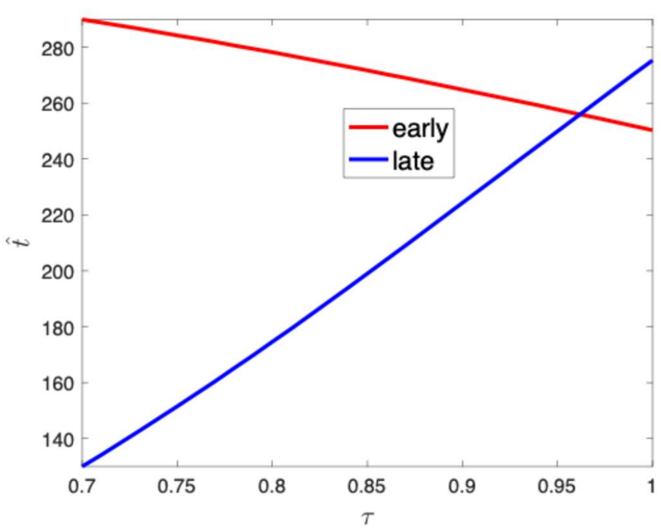
Comin-Lashkari-Mestieri productivity growth rates;
Employment Shares

Peak Time	**Peak M-Share**	**Peak Time Per Capita Income**
		

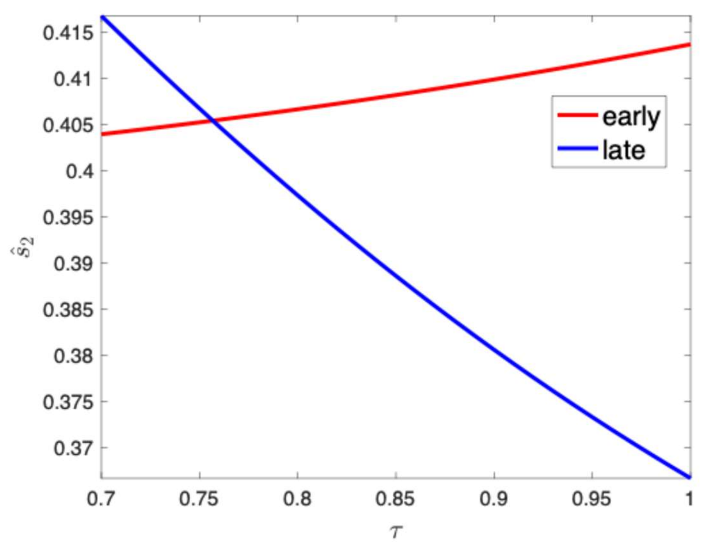
Reversal of $\hat{t}$ at $\tau \approx 0.96$, or $e^{\tau_1+\tau_2} \approx 2.295 \rightarrow \sqrt{2.295} \approx 1.515$ times higher in the importing country.
Reversal of $\hat{s}_2$ at $\tau \approx 0.77$ or $e^{\tau_1+\tau_2} \approx 1.947 \rightarrow \sqrt{1.947} \approx 1.395$ times higher in the importing country.

Comin-Lashkari-Mestieri productivity growth rates;
Value-Added Shares

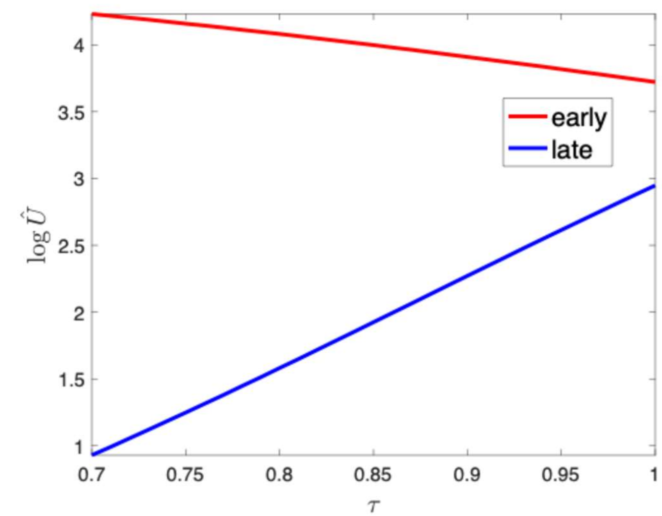
Peak Time



Peak M-Share



Peak Time Per Capita Income



Reversal of $\hat{t}$ at $\tau \approx 0.96$, or $e^{\tau_1+\tau_2} \approx 2.504 \to \sqrt{2.504} \approx 1.582$ times higher in the importing country.
Reversal of $\hat{s}_2$ at $\tau \approx 0.76$ or $e^{\tau_1+\tau_2} \approx 2.068 \to \sqrt{2.068} \approx 1.438$ times higher in the importing country.

3rd Extension: Introducing Catching Up

Narrowing a Technology Gap

We assumed that λ is time-invariant. This implies

The sectoral productivity growth rate is constant over time & identical across countries.
[In contrast, the aggregate growth rate, $g_U(t) \equiv U'(t)/U(t) = \sum_{k=1}^{3} g_k s_k(t)$, declines over time, $g_U'(t) = g_1 s_1'(t) + g_2 s_2'(t) + g_3 s_3'(t) = (g_1 - g_2) s_1'(t) + (g_3 - g_2) s_3'(t) < 0$, the so-called Baumol's cost disease.]

What if technological laggards can ***narrow a technology gap****, and hence achieve a higher productivity growth in each sector?*

Countries differ only in the *initial* value of lambda, $\boldsymbol{\lambda_0}$, converging exponentially over time at **the same rate,**

$$A_j(t) = \bar{A}_j(0) e^{g_j(t - \theta_j \lambda_t)}, \quad \text{where} \quad \lambda_t = \lambda_0 e^{-g_\lambda t}, \quad g_\lambda > 0.$$

$$\Rightarrow \frac{1}{s_2(t)} = \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{\alpha[(\theta_1 g_1 - \theta_2 g_2)\lambda_t - (g_1 - g_2)t]} + 1 + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{\alpha[(\theta_3 g_3 - \theta_2 g_2)\lambda_t + (g_2 - g_3)t]}$$

Again, by setting the calendar time such that $\hat{t}_0 = 0$ for the frontier country with $\lambda_0 = 0$,

Peak Time

$$\hat{t} = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3}\lambda_{\hat{t}} + D(g_\lambda \lambda_{\hat{t}})$$

Peak Share

$$\frac{1}{s_2(\hat{t})} = 1 + \left(\frac{\tilde{\beta}_1 + \tilde{\beta}_3}{\tilde{\beta}_2}\right)\left[\frac{(g_2 - g_3)e^{a(g_2 - g_1)D(g_\lambda \lambda_{\hat{t}})} + (g_1 - g_2)e^{a(g_2 - g_3)D(g_\lambda \lambda_{\hat{t}})}}{g_1 - g_3}\right]\left[e^{\frac{a(g_1 - g_2)(g_2 - g_3)}{(g_1 - g_3)}}\right]^{\left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} + \frac{\theta_3 g_3 - \theta_2 g_2}{g_2 - g_3}\right)\lambda_{\hat{t}}}$$

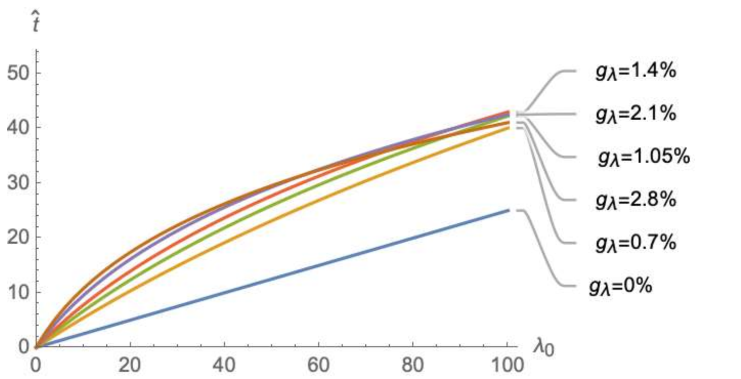
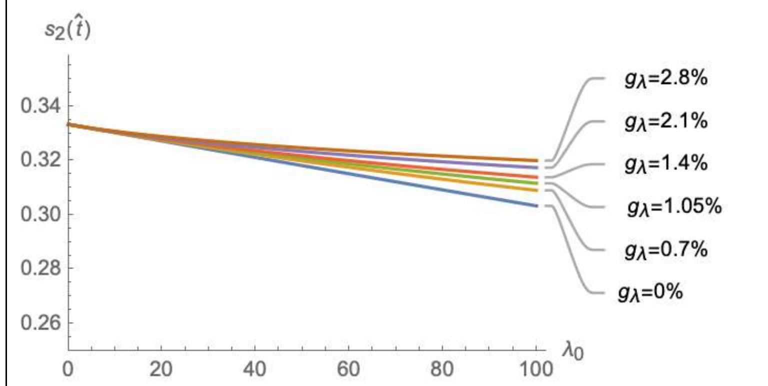
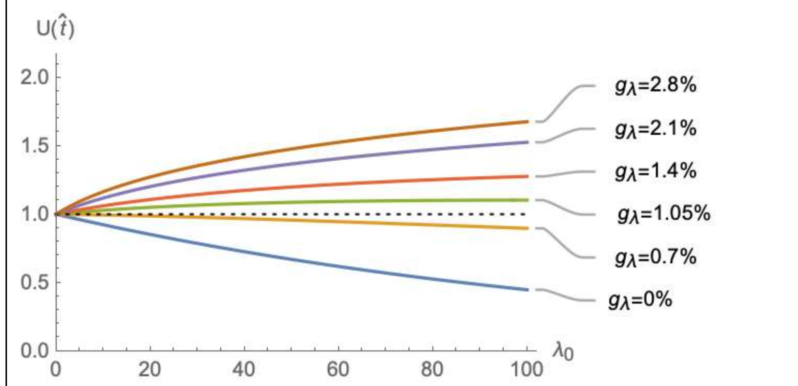
Peak Time Per Capita Income

$$U(\hat{t}) = \left\{\left(\tilde{\beta}_1 e^{-a g_1 D(g_\lambda \lambda_{\hat{t}})} + \tilde{\beta}_3 e^{-a g_3 D(g_\lambda \lambda_{\hat{t}})}\right)e^{-a\frac{(\theta_1 - \theta_3)g_1 g_3}{g_1 - g_3}\lambda_{\hat{t}}} + \left(\tilde{\beta}_2 e^{-a g_2 D(g_\lambda \lambda_{\hat{t}})}\right)e^{-a\frac{(\theta_1 - \theta_2)g_1 g_2 + (\theta_2 - \theta_3)g_2 g_3}{g_1 - g_3}\lambda_{\hat{t}}}\right\}^{-\frac{1}{a}}$$

where

$$D(g_\lambda \lambda_{\hat{t}}) = \frac{1}{a(g_1 - g_3)}\ln\left[\left(\frac{g_1 - g_2 + (\theta_1 g_1 - \theta_2 g_2)g_\lambda \lambda_{\hat{t}}}{g_2 - g_3 - (\theta_3 g_3 - \theta_2 g_2)g_\lambda \lambda_{\hat{t}}}\right)\left(\frac{g_2 - g_3}{g_1 - g_2}\right)\right].$$

For $g_\lambda = 0, D(g_\lambda \lambda_{\hat{t}}) = D(0) = 0$, and all the parts in red disappear, and we go back to the baseline model.

Peak Time	Peak M-Share	Peak time Per Capita Income
		

Technological laggards

- peak later in time,
- have lower peak M-shares
- have lower peak time per capita income, unless g_λ is too large: Comin-Mestieri (2018)

Concluding Remarks

A Parsimonious model of Rodrik's (2016) PD based on

- **Differential productivity growth rates across complementary sectors**, as in Baumol (67), Ngai-Pissarides (07).
- **Countries heterogeneous only in their technology gaps**, as in Krugman (1985).
- Sectors differ in the extent to which technology gap affects their adoption lags, unlike in Krugman (1985)

We find that PD occurs for

- cross-country productivity difference larger in A than in S.
- technology adoption takes not too long in M.
- Technology adoption takes longer in S than in A.

which implies that cross-country productivity difference the largest in A.

The baseline model assumes **homothetic CES, no international trade, no catching up.**

In three extensions, we showed that the results are ***robust*** against introducing

- **The Engel effect** with income-elastic S & income-inelastic A, using nonhomothetic CES: CLM(21), Matsuyama(19)

The Engel effect changes the shape of the time paths, but not the implications on technology gaps on PD.
The Engel effect *alone* could not generate PD w/o counterfactual implications on cross-country productivity differences

- **International trade in A and in M**, but PD becomes weaker.
- **Narrowing a technology gap** to allow technological laggards to catch up,

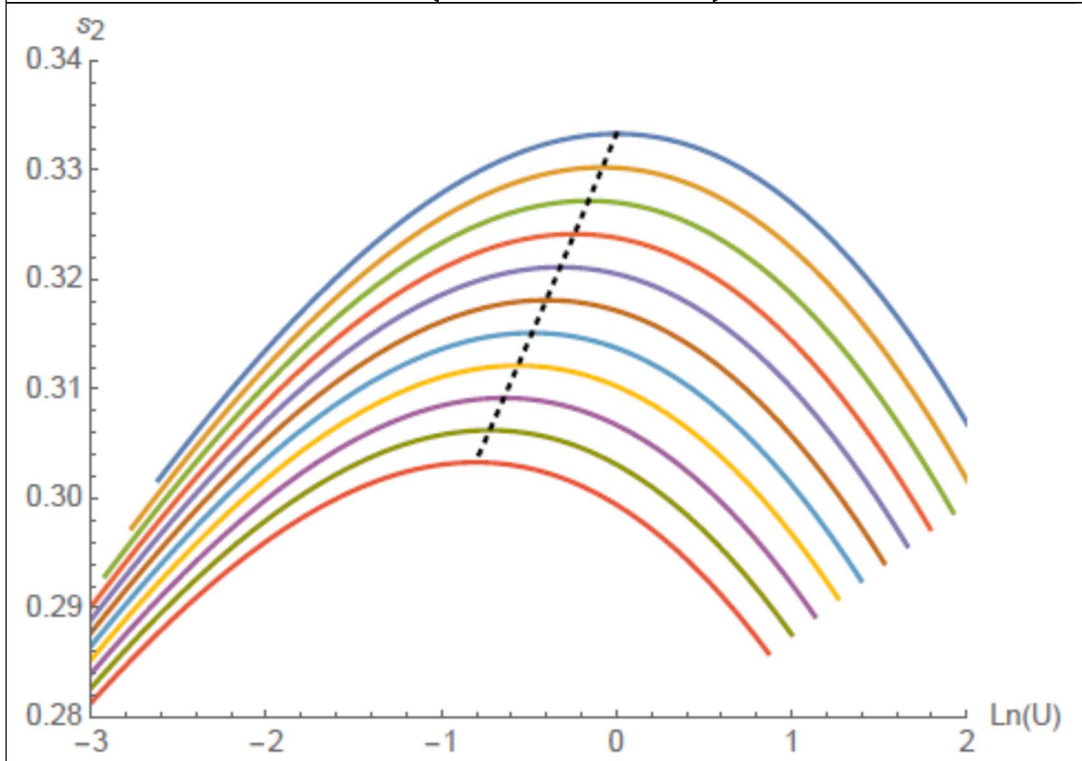
unless the catching-up speed is too large.

Appendix

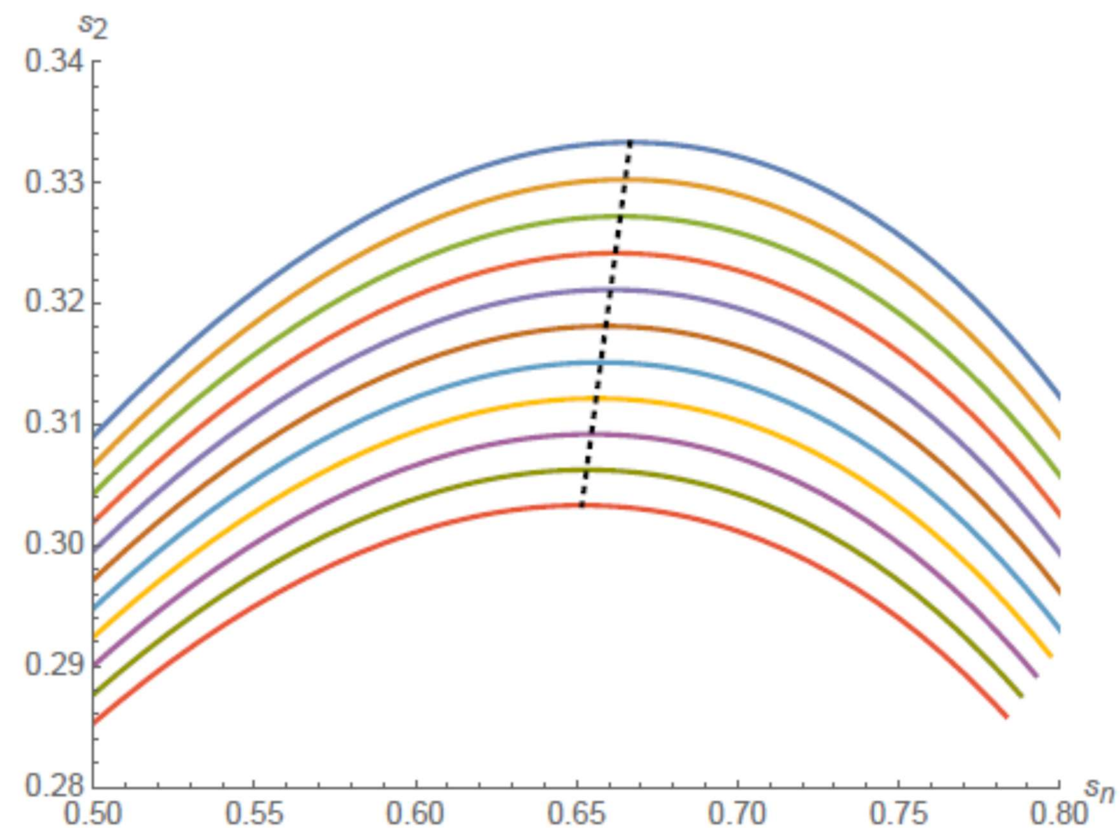
Appendix: Non-agricultural share as another measure of development, $1 - s_1(\hat{t}) = s_2(\hat{t}) + s_3(\hat{t}) \equiv s_n(\hat{t})$

Baseline Homothetic Case:

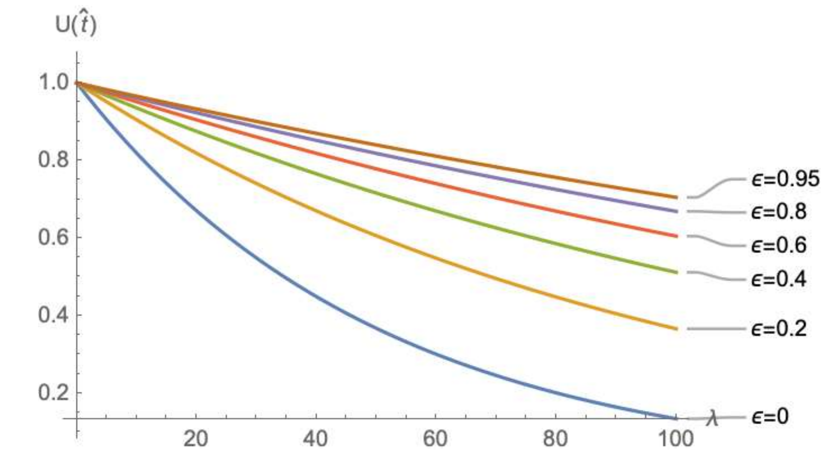
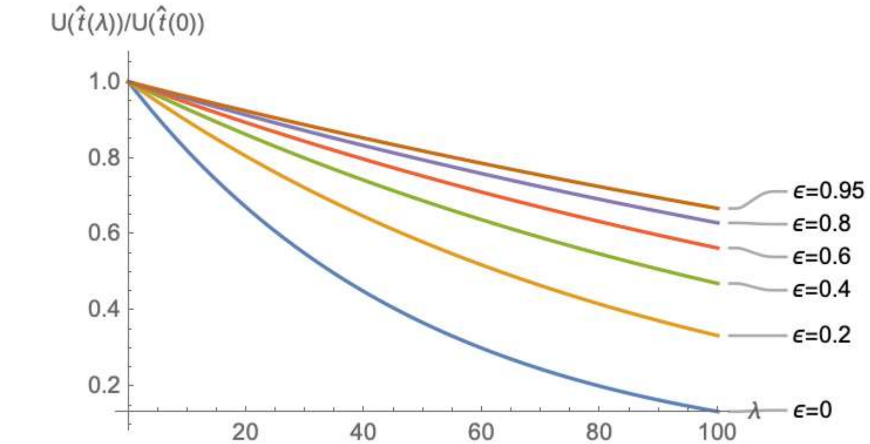
$\left(\ln U(t), s_2(t)\right)$



$\left(s_n(t), s_2(t)\right)$



Nonhomothetic Cases:

<table>
<tr><td></td><td>Unbiased: $\varepsilon_1 = 1 - \epsilon < \varepsilon_2 = 1 < \varepsilon_3 = 1 + \epsilon$</td><td>Biased: $\varepsilon_1 = 1 - \epsilon < \varepsilon_2 = 1 + \epsilon/3 < \varepsilon_3 = 1 + 2\epsilon/3$</td></tr>
<tr><td>$\ln U(t)$</td><td>
</td><td>
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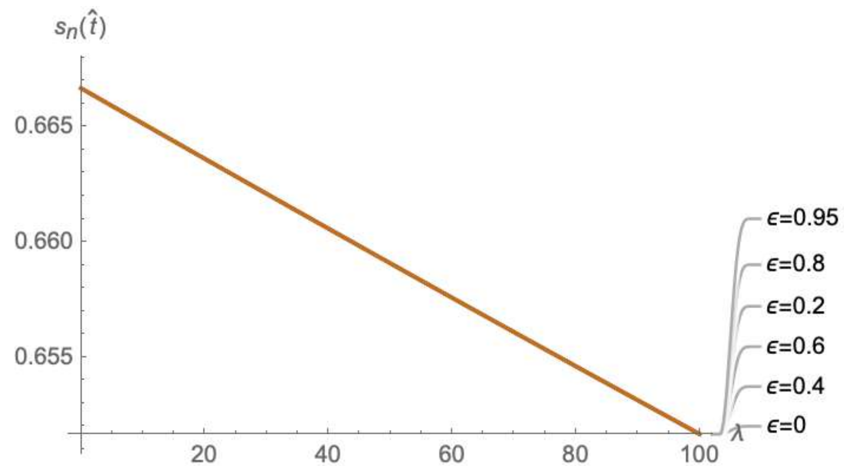
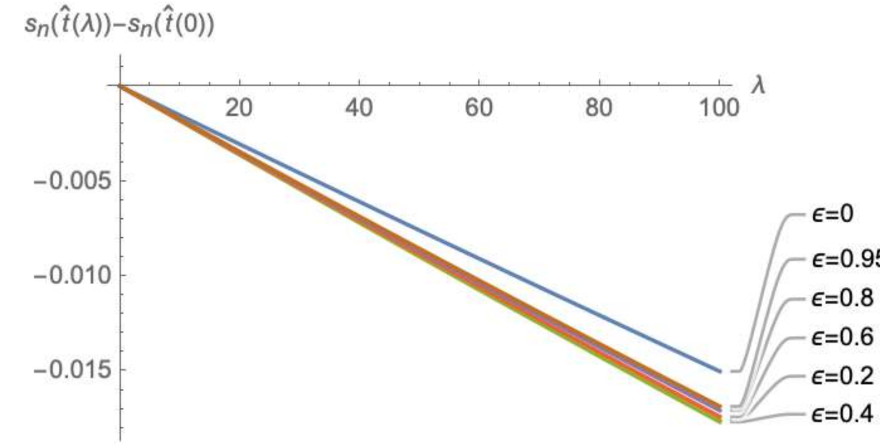
<tr><td>$s_n(t)$</td><td>
</td><td>
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In the biased case, the frontier country's peak values are affected by ϵ.